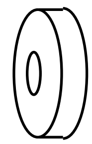
Making and Using the “Roller Coaster Simulator” for Education

by Tony Wayne

This is a demo that mounts on a blackboard that is magnetic. This demo will allow the teacher to demonstrate Galileo’s inertia experiment and how and why the hills on roller coasters are designed the way they are. When it is built, it will be an adjustable rail road type track.

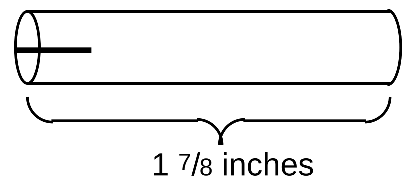
Materials:

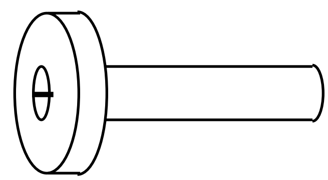
- 12 feet of vinyl tubing with an outside diameter of 1/4 inch. Some larger hardware stores sell the tubing. (20¢ per foot)
- 18- 1 1/8 inch diameter magnets from Radio Shack.
- 37 1/2 inches of dowel rod 3/8 inches in diameter .
- 1 small tube of contact cement
- 1 tube of “Household” GOOP® or any other adhesive that will glue vinyl. (Epoxy will not work.)



STEP 1

Cut the dowel rod into 20 pieces 1 7/8 inches long. Cut a 1/4 deep slit into one end of each dowel rod. The slit should be about 1/16 inch wide. A band saw is a good tool for doing this.



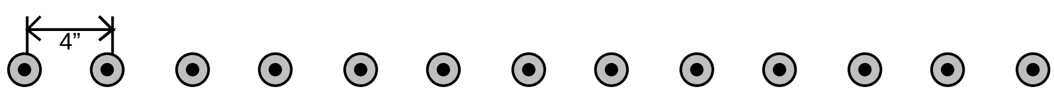


STEP 2

Insert the slit end of each dowel into a magnet.

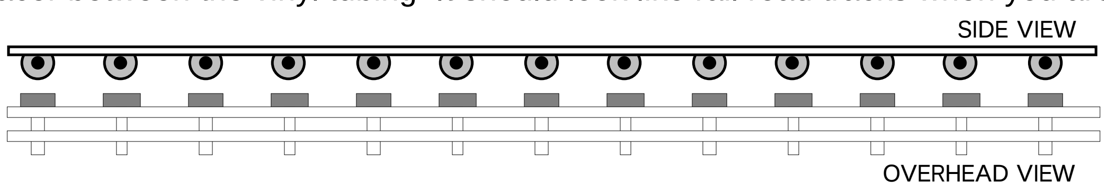
STEP 3

Cut some wood into 1/4” X 1/4” X 1 1/2” RECTANGLES. These will be used as spacers. Mark the tubing every 4 inches with a permanent marker. Place the magnets on the board in a straight line 4 inches apart from each other.



STEP 4

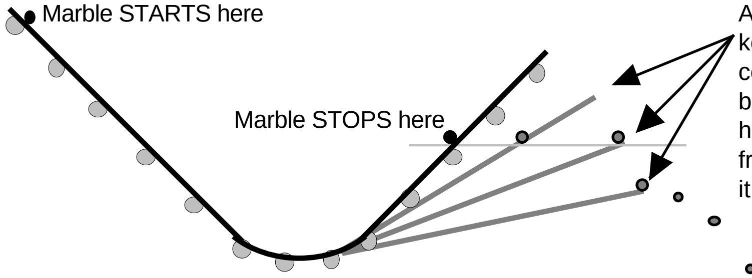
Put some contact cement on the top side of each dowel rod. Put some more contact cement on the vinyl tubing at each mark. With another person’s help, lay the tubing against the magnets on top of the dowel. When you have laid out 18 magnets, 6 feet, stop. Cut the tubing. Lay the other piece of tubing 1/4 inch away from the first piece. Use the cut wood as a spacer between the vinyl tubing. It should look like rail road tracks when you are done



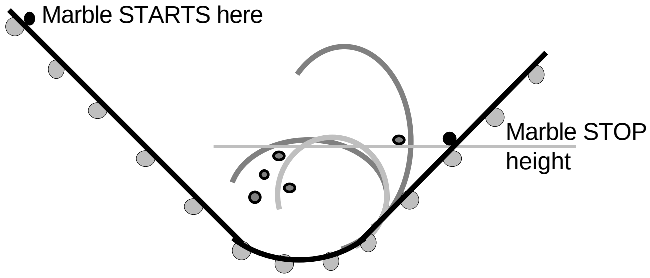
STEP 5

Final step. The track’s glue joints are weak. Reinforce them with GOOP™ on the underside. Be sure not to get any glue on top of the tubing. A little piece of glue on top of the tubing will cause the marble to roll off it.

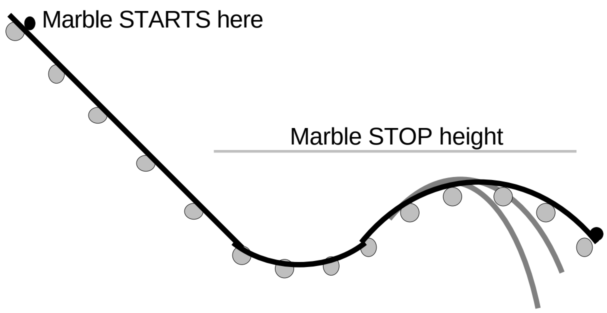
USING THE ROLLER COASTER SIMULATOR



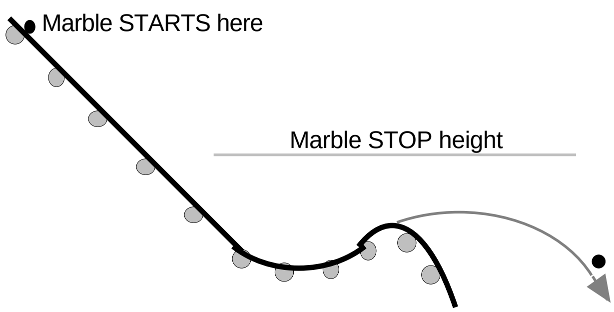
Adjust the ramp to lower angles. The marble keeps rolling to the same height. Galileo concluded from a similar experiment that the ball will keep rolling until it reaches the same height it started from if there were no friction. (With friction, it will keep rolling until it reaches the stop height.)



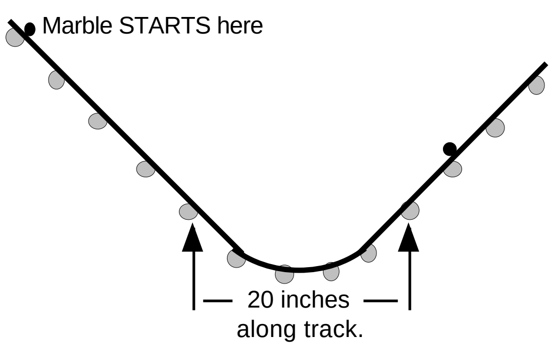
Adjust the ramp to different loop radii. Notice if the loop's radius is too big, the marble falls off. Try adjusting the radius until the ball makes it around the loop.



Adjust the ramp to different hill shapes. Drop the marble from the same height each time. Keep adjusting the hill's shape until the marble no longer leaves the track. This is the desired hill shape for that start height. After all, if the roller coaster left the tracks each time, the company would lose its customers. (Try a lower start height and see how the hill shape would change.)



Adjust the ramp to make a gradual climb on the left side and steep drop on the right. Trace the path the bottom of the ball makes as it leaves the track. Line the track up with the traced curve. The ball will stay on the track. A coaster's hills are designed to give the rider the weightless feeling they would get if the track were not there.



PRACTICE SPEED ESTIMATION

Each support is 4 inches apart. To estimate the velocity at the bottom of this curve, count an equal number of supports on each side of the dip's center. Calculate this distance. Time how long the ball takes to travel the distance. Calculate the average speed, (distance/time). The average will equal the exact speed at the center of the dip -but only if (1) the distance measured is equal on both sides of the dip's center and (2) the region measured is fairly symmetrical in shape.

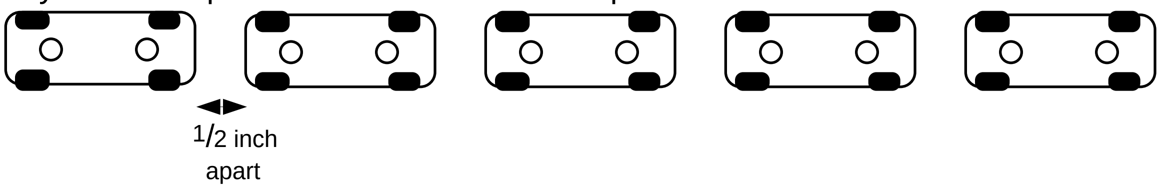
Making a Roller Coaster Train from Hot Wheels™

Hot Wheels™ is a registered trademark of Mattel, Inc.

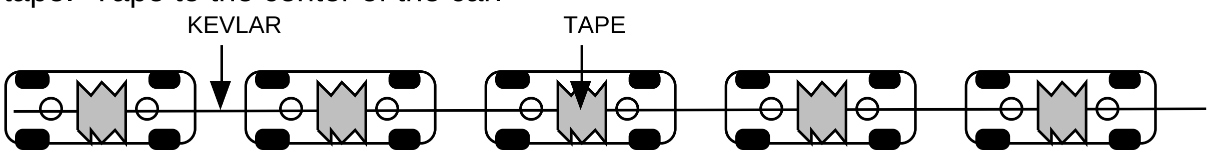
MAKING A HOT WHEELS™ TRAIN

MATERIALS:
5 -Hot Wheels™ cars. (Use cars with a short front to back wheel base.)
24 inches -Kevlar. (This can be found in most fishing goods departments of larger stores with the fishing line.)
30 minute Epoxy

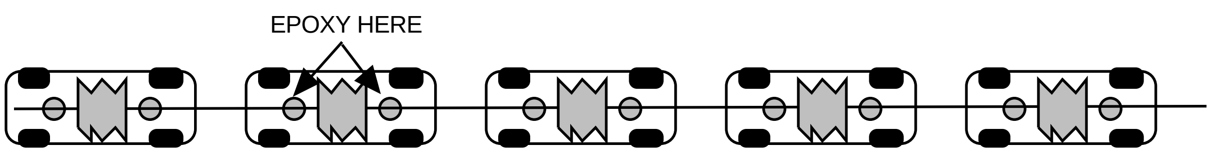
PROCEDURE:
1 Lay the cars upside down about 1/2 inch apart from each other.



2 Lay the Kevlar from front to back on the row of cars. Tape the Kevlar in place with masking tape. Tape to the center of the car.



3 Mix and apply the epoxy over the Kevlar and at each car's rivet. The cars have two rivets on the undercarriage. Let dry for at least 30 minutes. If the epoxy mix is not right, the cars may have to dry longer.



4 To aide in seeing the center of mass of the train of cars, paint the middle car. It is located at the center of mass.

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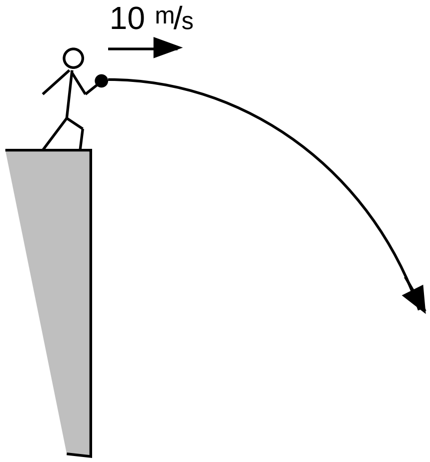
Roller Coaster Activities for the Classroom

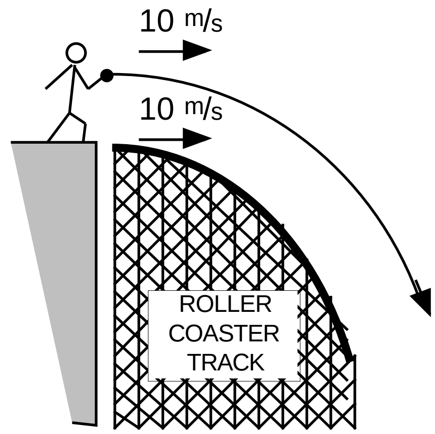
(This Section Contains Worksheets, Roller Coaster Design labs and Tests, Various Roller Coaster Labs and Worksheet Answers)

HILL DESIGN ACTIVITY

Stand on the edge of a cliff and throw a ball horizontally. It would travel as shown below.



This is the path an object would take if it were weightless.



The roller coaster track is designed to have the same arc shape as a ball that is thrown off a cliff.

Because the two paths are the same, it stands to reason that the roller coaster car would also be weightless as it travels down the hill.

HILL DESIGN: ACTIVITY 1

Hold a large piece of plywood at a 60° angle to the floor. Roll a WET tennis ball horizontally across the top of the board. Look at the water trail it left behind. Roll a WET tennis ball FASTER horizontally across the top of the board. Look at the new water trail it left behind.

Draw the results below.

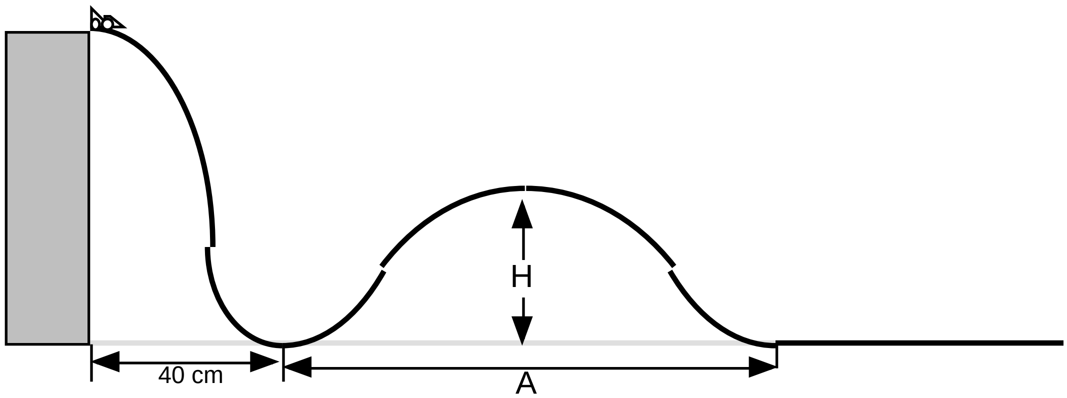
SLOW BALL'S PATH	FAST BALL'S PATH

HILL DESIGN: ACTIVITY 2

Materials: HotWheels™ Track, Hot Wheels™ car

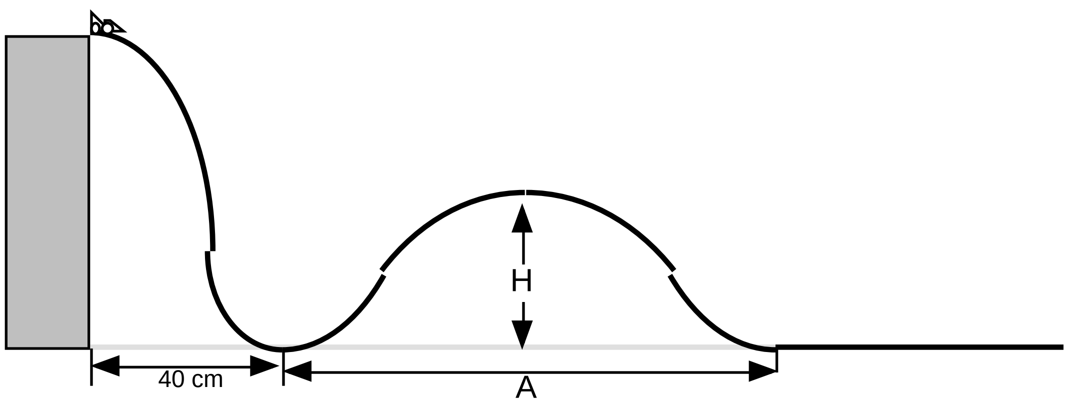
Rest the top of the track on the edge of the counter. Make the track's contact point with the ground 40 cm away from the edge on the floor. Using books:

1. Create a hill that when the car travels over it, the car becomes airborne on the far side. Write down the measurements shown below.



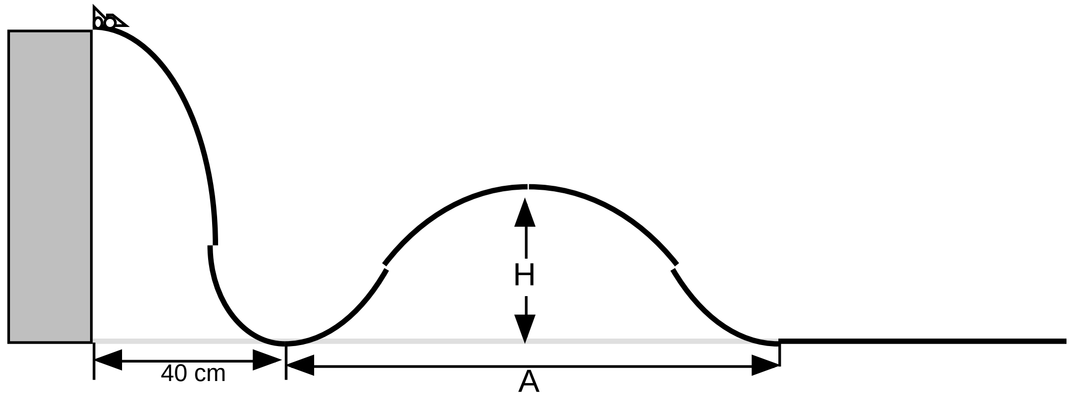
Distance A: _________meters
Distance H _________meters

2. Create a hill that when the car travels over it, the car barely becomes airborne on the far side and it still lands on the track on the way down. Write down the measurements shown below.



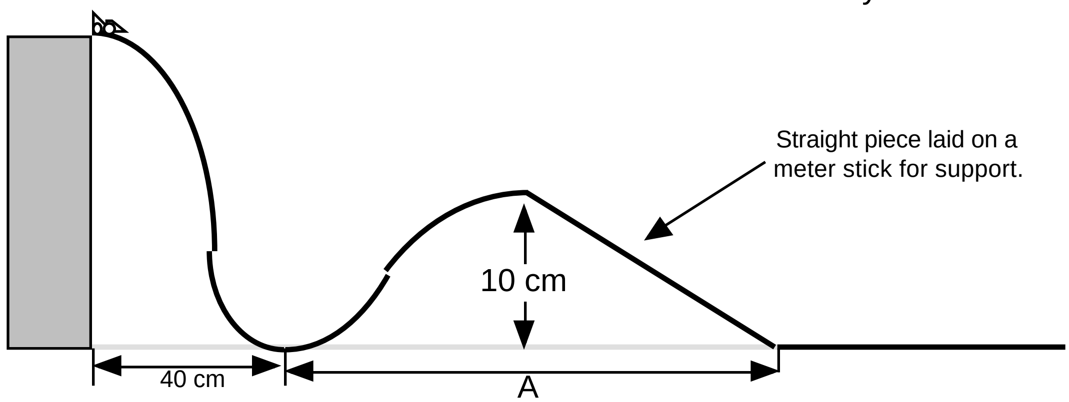
Distance A: _________meters
Distance H _________meters

3. Create a hill that when the car travels over it, the car never becomes airborne on the far side. Write down the measurements shown below.



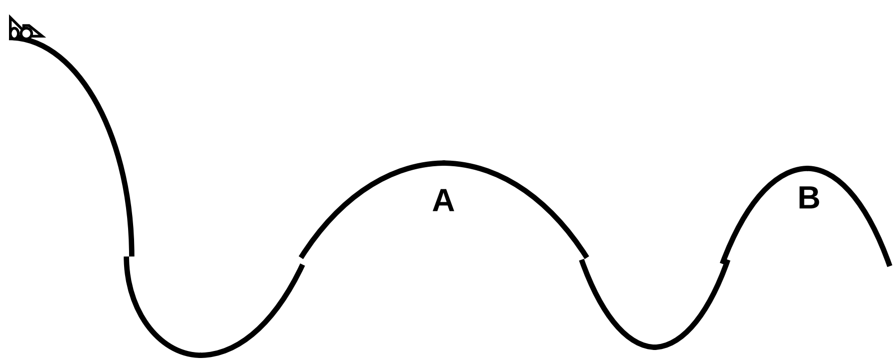
Distance A: _________meters
Distance H _________meters

4. Will the car come off the far side of the track shown below? Why?



Distance A: _________meters

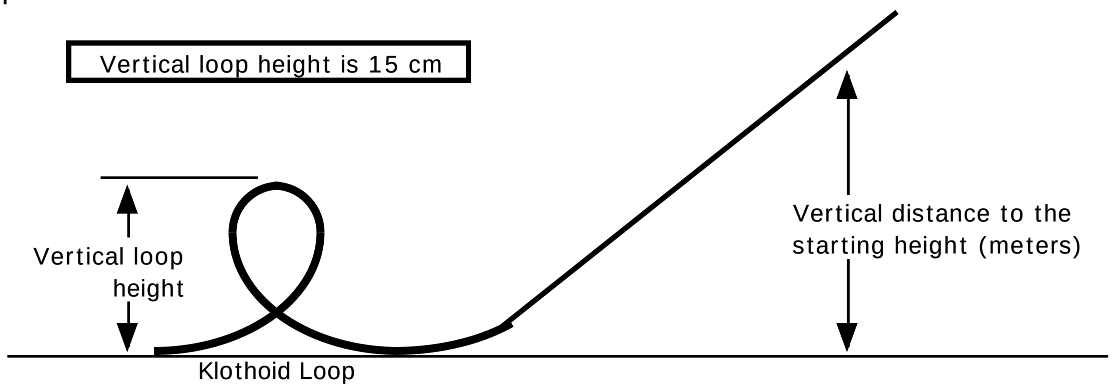
5 **Over which hill will the car most likely come off the track? Why?**



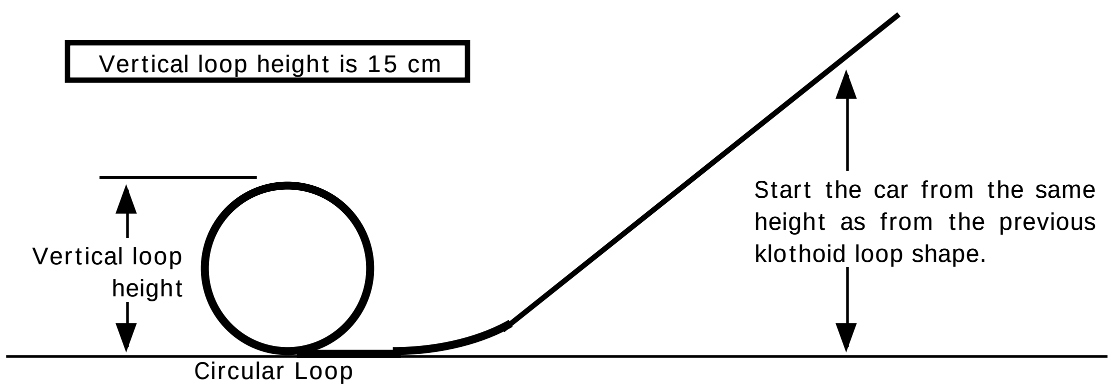
LOOP DESIGN: ACTIVITY 1

Materials: HotWheels™ Track, Hot Wheels™ car, meter stick

Construct a loop in the shape of a "klothoid." The klothoid shape is like an upside down tear drop.



Start the car at various heights. Determine the starting height in which the car just makes it around the loop without falling off the track. This height is ___________m.



Does the car make it around the circular loop from the same height as before? ______ If the car falls off the track, draw an arrow on the circular loop to indicate where this happens.

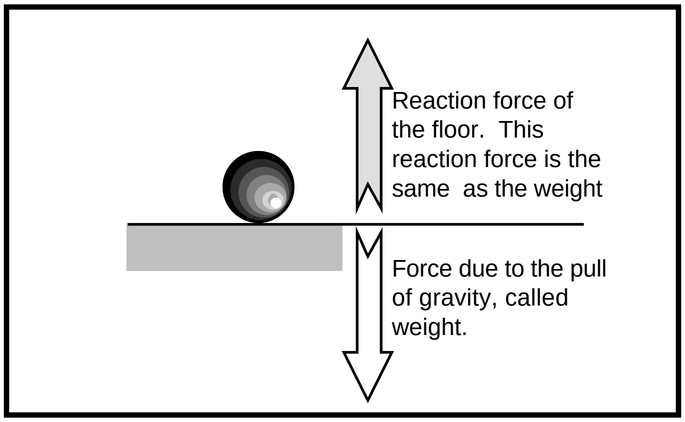
What is the minimum height the car needs to be started from to just make it around the circular loop?

_________________________m.

Why are loops on most roller coaster rides klothoid shaped instead of circular?

FREE FALL (Weightlessness)

In order to feel weight there must be a "reaction" force.



If you jump out of a window, you would feel weightless because there is no reaction force pushing up on your feet.

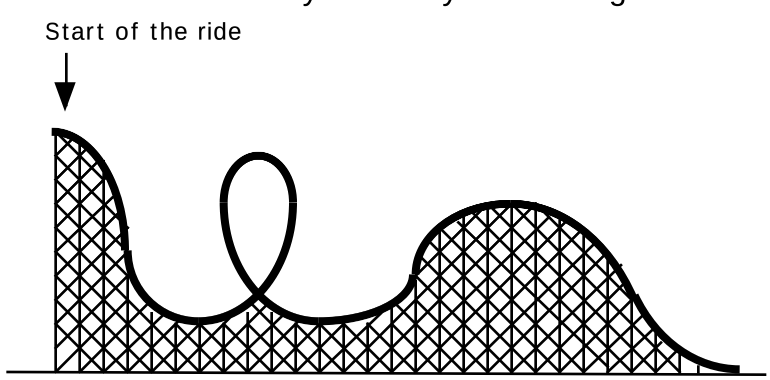
FREE FALL (Weightlessness): ACTIVITY 1

Materials: cup (with holes), water, chair, paper towels, trash can

Poke two holes on opposite sides of the bottom of the cup. Stand in a chair. Hold the cup with your fingers over the holes. Fill the cup 1/3 full with water. Hold the cup with your other hand. Briefly remove your fingers from the holes. Observe what happens. Cover the holes again with your fingers. Hold the cup up as high as possible. Drop the cup and observe what happens to the water this time.

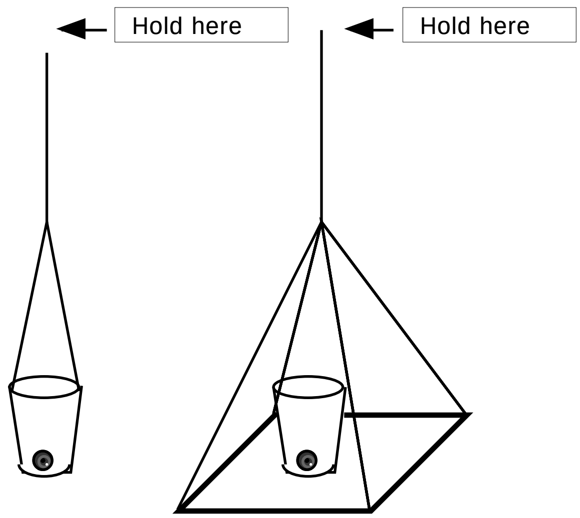
Why doesn't the water come out of the holes when it is dropped?

When on the roller coaster below do you think you are weightless?



STAY SEATED

Swing the apparatus around with a super ball in the cup. **STAND CLEAR OF ANYONE ELSE AROUND YOU.** Swing it just slow enough so the ball does not fall out.



Use either apparatus

Take whatever measurements you need to calculate the centripetal acceleration of the ball in the cup. ____________.

How many g's is this? _________g's.

PRACTICING YOUR "ESTIMATIONS"

Mark a starting point and walk 10 steps. Walk naturally. Measure this distance. Calculate the distance traveled for each of your steps.

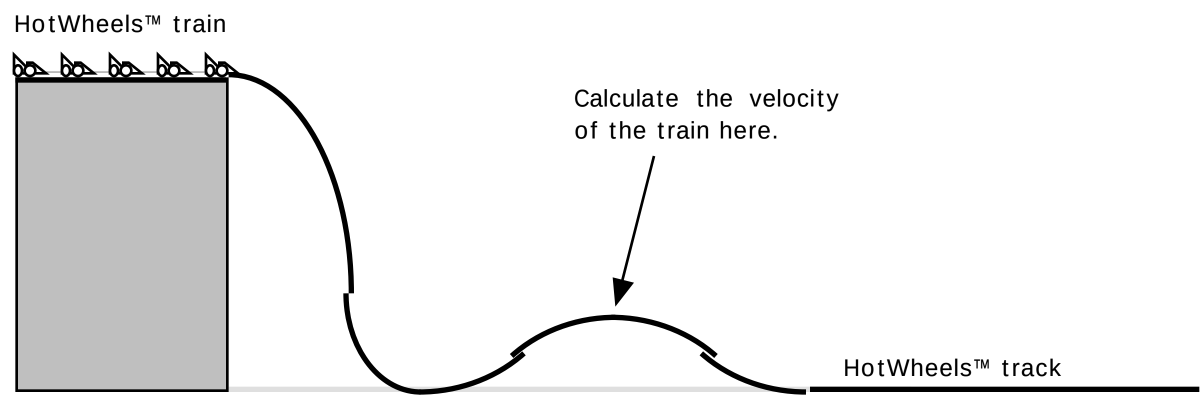
$\frac{\text{Distance}}{\text{Step}}$ = _________

Using the previous information and "Two Angle Method", calculate the height of the edge of the school building. **WE WILL ALL GO OUTSIDE TOGETHER FOR THIS... WAIT.**

Building edge height ___________.

MEASURING THE VELOCITY OF A MOVING OBJECT

In the classroom is a HotWheels™ set up. On this set up, measure the velocity of the HotWheels™ train as it passes over the second hill. The length of the train is the distance used to calculate the average velocity. The time measured is the time for the entire train to pass one point on the center of the hill.Treat this as a normal lab and repeat the process a number of times and take the average of the trials.



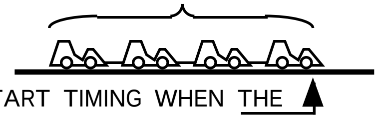
HotWheels™ train velocity: _____________

NOTE: The train is heavier than a single car. When the train is in motion, it will cause the track to slide around. Hold the track down firmly to prevent accidents.

On the following 2 pages are handouts for students to use with the coaster activities or to take with them to the amusement park.

MEASURING VELOCITY AT ONE LOCATION ON THE RIDE

CAR LENGTH

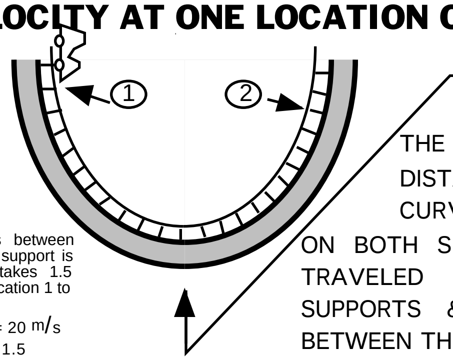


START TIMING WHEN THE FRONT OF THE CAR PASSES A POINT. STOP TIMING WHEN THE END OF THE CAR PASSES THE POINT.

EXAMPLE

There are 20 supports between locations 1 and 2. Each support is 1.5 meters apart. It takes 1.5 seconds to travel from location 1 to 2.

Velocity = $\frac{20(1.5)}{1.5} = \frac{30}{1.5} = 20\ ^{m}/_{s}$



TIME THE DISTANCE FROM POINT 1 TO POINT 2. THE POINTS MUST BE THE SAME DISTANCE FROM THE CENTER. THE CURVE'S SHAPEMUST BE THE SAME ON BOTH SIDES. ESTIMATE THE DISTANCE TRAVELED BY COUNTING THE STEEL SUPPORTS & ESTIMATING THE DISTANCE BETWEEN THEM.

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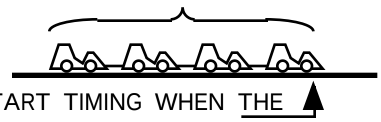

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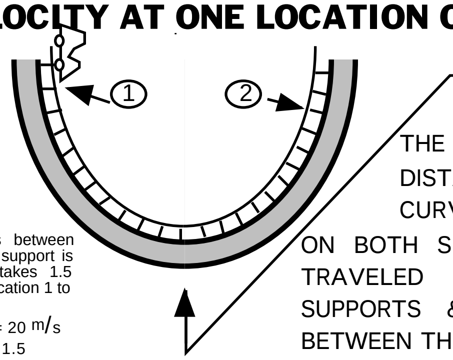

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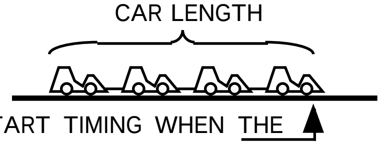

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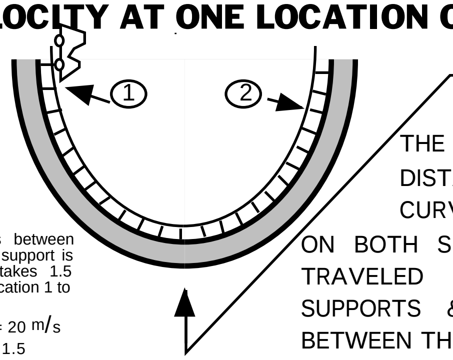

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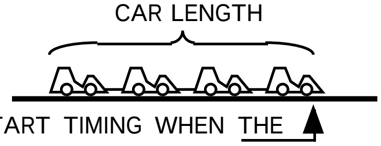

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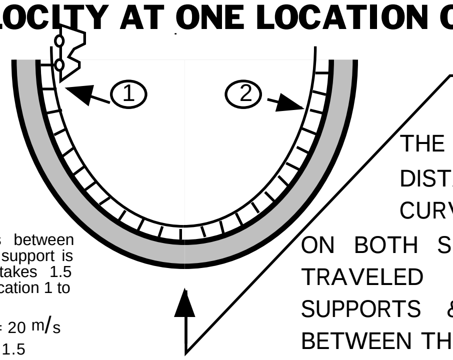

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FORMULAE AND CONSTANTS

$x = x_o + v_o t + (1/2)at^2$

$v = v_o + at$

$v^2 = v_o^2 + 2ax$

$v_{avg} = \frac{v+v_o}{2} = \frac{x}{t}$

$F = ma$

$p = mv$

$E_K = (1/2)mv^2$

$U_g = mgh$

$U_s = (1/2)kx^2$

$W = F_d d$

$E_{TOTAL} = E_K + U_g + U_s$

$P = W/t$

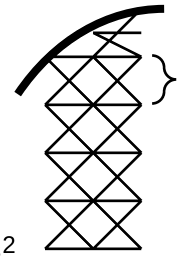
$P = Fv$

$T = 2\pi\sqrt{\frac{L}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$g = 9.80\ m/s^2$

$g = 32.15\ ft/s^2$



Estimate this distance and count support structure up for the height and width.

If this structure is 10 ft high, then the right support is 5 x 10ft = 50 ft high.

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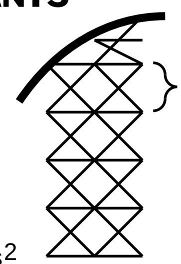

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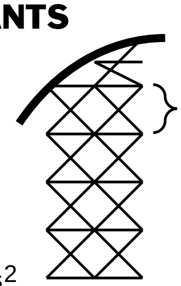

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$E_K = (1/2)mv^2$

$U_g = mgh$

$U_s = (1/2)kx^2$

$W = F_d d$

$E_{TOTAL} = E_K + U_g + U_s$

$P = W/t$

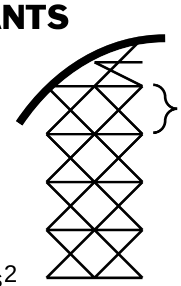
$P = Fv$

$T = 2\pi\sqrt{\frac{L}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$g = 9.80\ m/s^2$

$g = 32.15\ ft/s^2$

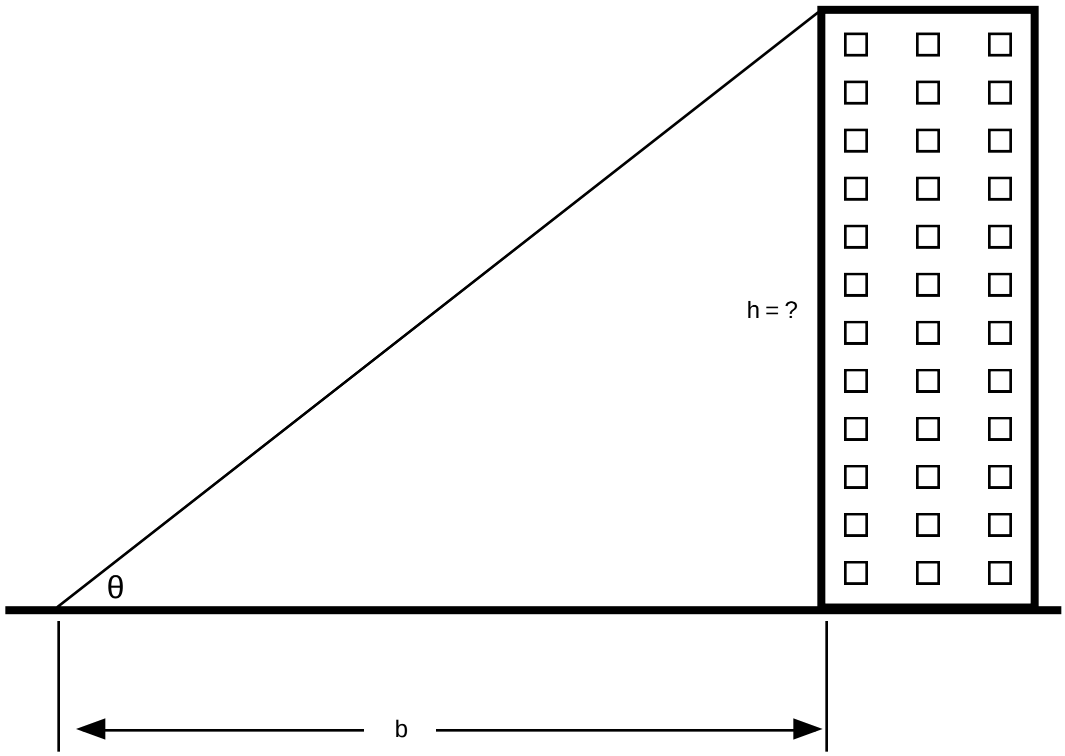


Estimate this distance and count support structure up for the height and width.

If this structure is 10 ft high, then the right support is 5 x 10ft = 50 ft high.

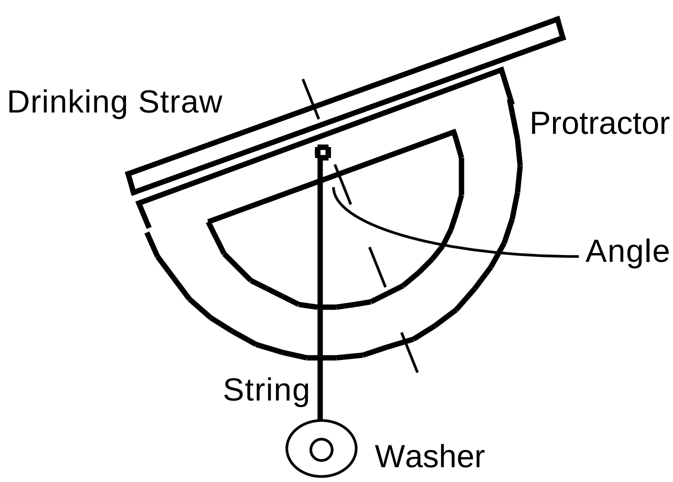
Height: Single Station Method

This is good method to use when you are on flat, level, ground and you know the distance from where you are standing to the edge of the object whose height you are trying to calculate.



$$\tan(\theta) = \frac{h}{b}$$

$$\boxed{b = \frac{h}{\tan(\theta)}}$$

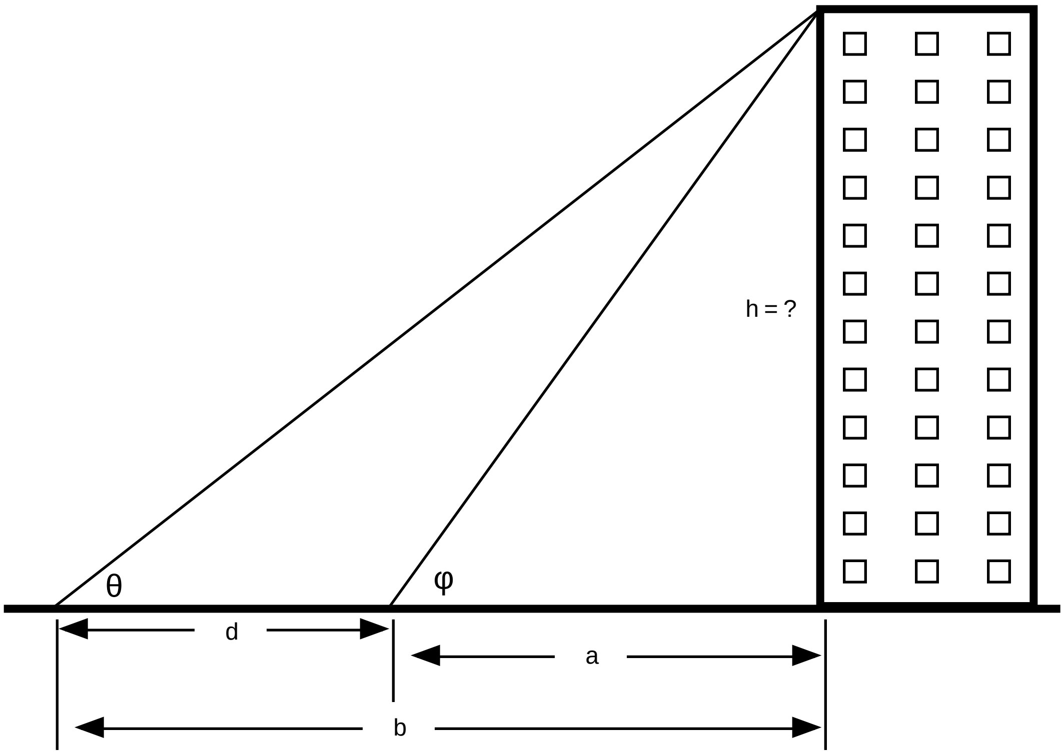


Look at the point of interest through the drinking straw. Read the angle.

Don't forget to add the height from the ground to your eye to your measurements.

Height: 2 Station Method

This is a good method when you do not know the distance between you and the structure. All you will need to do is take two angle measurements of the structure and measure the distance between the measurements as shown in the diagram below.



$$\tan(\theta) = \frac{h}{b} \qquad \tan(\varphi) = \frac{h}{a}$$

$$b = \frac{h}{\tan(\theta)} \qquad a = \frac{h}{\tan(\varphi)}$$

$$d = b - a$$

$$d = \frac{h}{\tan(\theta)} - \frac{h}{\tan(\varphi)}$$

$$d = \left[\frac{1}{\tan(\theta)} - \frac{1}{\tan(\varphi)}\right]h$$

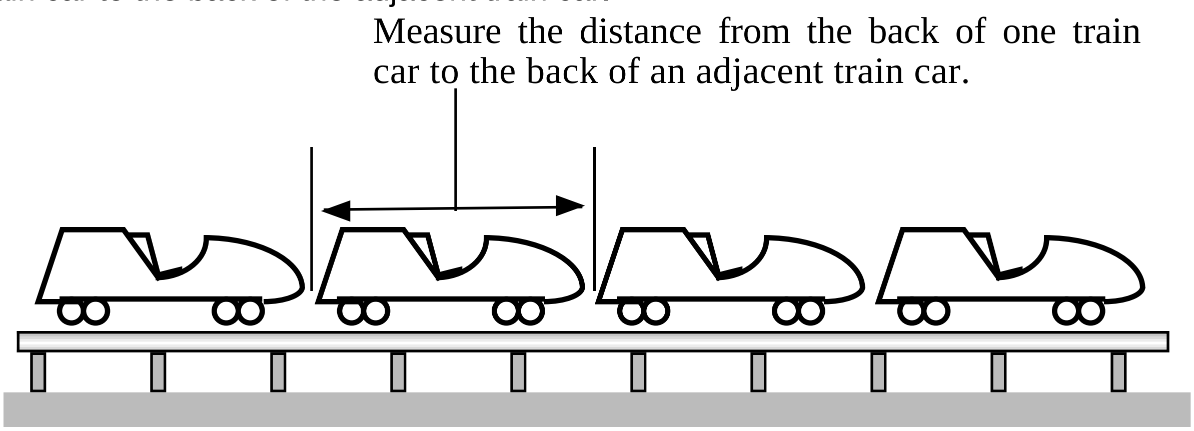
$$d = \left[\frac{1}{\tan(\theta)} - \frac{1}{\tan(\varphi)}\right]h$$

$$\boxed{h = d[\tan(\theta) - \tan(\varphi)]}$$

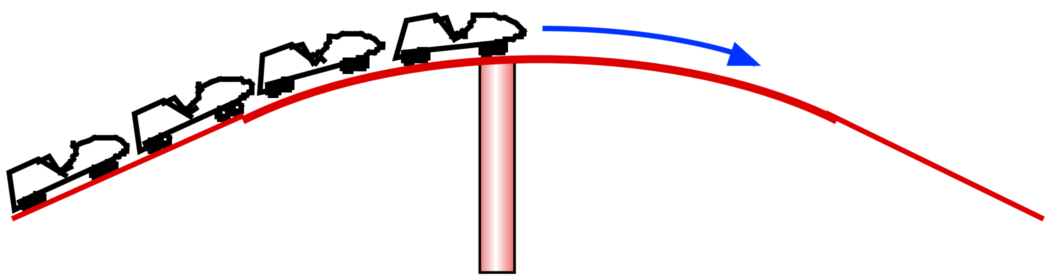
VELOCITY: Length of train method

Average velocity equals distance over time.

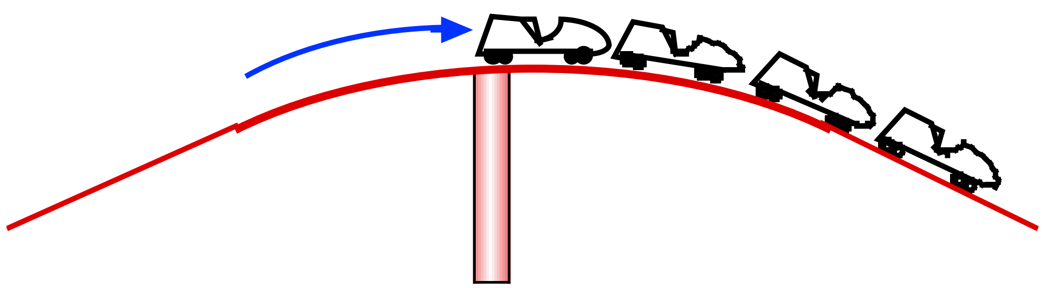
The distance measurement comes from the train of cars. While in the loading station, measure the length of the train. This can be done quickly by measureing from the back of one train car to the back of the adjacent train car.



Then multiply this distance by the number of cars in the train.



Time how long it takes for the front and the back of the train to pass the same point on the tracks. Pick a point that is in the very top of a hill or the very bottom of a dip.

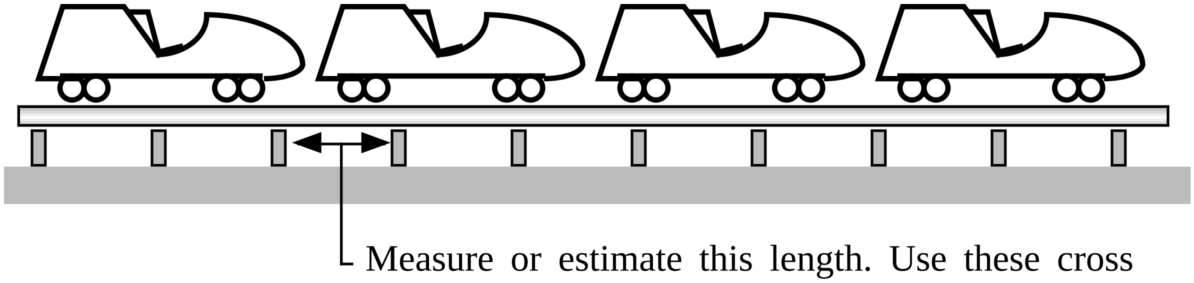


$$V_{average} = \frac{\text{length of train}}{\text{time to pass one location}}$$

VELOCITY: Length of track method

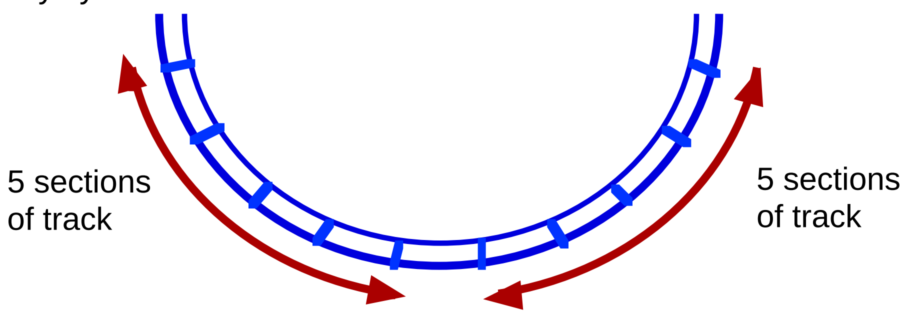
Average velocity equals distance over time.

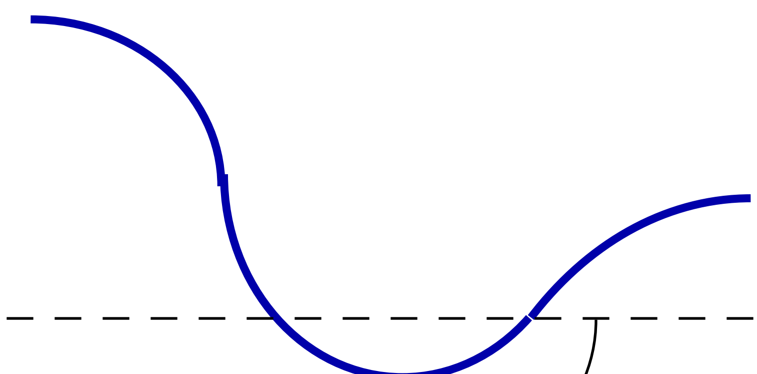
The distance measurement comes from a known length of track. Most tracks are constructed from track cross ties that are some equal distance apart from each other. Either get this distance form the park, measure or estimate it.



Measure or estimate this length. Use these cross tie supports and their distance as a measuring stick on the track.

Time how long it takes for the middle of the train to pass across all 10 sections. You may use any number of track sections so long as there are an equal number on each side and shape of the track is fairly symmetrical about the middle section of the track.





Cross tie supports above this dotted line are not symmetrical about the center of the dip.

$$V_{average} = \frac{\text{length of distance travled}}{\text{time to cover this length}}$$

You are a secret agent from a competing amusement park. Your task is to take a roller coaster and find out EVERYTHING you can about the roller coaster.

NEATNESS COUNTS!

YOUR ROLLER COASTER IS THE

____

Group Member's

____

____

____

____

____

PART 1 Most roller coasters wrap themselves in a circle. Draw a **SCALE** drawing of your roller coaster's first **5 peaks and dips** if you could straighten it out. (You do not have to use all the horizontal length of this graphing space -the numbers are just for reference.)

START 1 2 3 4 5 6 7

8 9 10 11 12 13 14 15

16 17 18 19 20 21 22 23

- Label each **hill's peak as a letter, A - E** on your scale drawing. You do not have to use every letter. Note, the top of a loop counts as a hill peak.
- Label each **dip as a number, 1 - 5** on your scale drawing. You do not have to use every number. Note, each time the roller coaster is at the bottom of a loop counts as a different dip.

PART 2 Calculate the height at each peak.

PEAKS

A ________ **B** ________ **C** ________ **D** ________ **E** ________

PART 3 Calculate the velocity at each letter and number.

PEAKS

A ________ **B** ________ **C** ________ **D** ________ **E** ________

DIPS

1 ________ **2** ________ **3** ________ **4** ________ **5** ________ **6** ________

PART 4 Someone in your group needs to ride the roller coaster. Measure the g's at the first 4 dips and hills. Note which part of the car you are riding in at the time of the measurement.

	g's	**Number of rows of seats from the front** (middle is best)
1	________	________________________________
2	________	________________________________
3	________	________________________________
4	________	________________________________

PART 5 Measure the g's at the first 2 dips and the first 2 peaks after the initial hill from the back, middle and front of the train.
For each location indicate how many seats from the front you are riding.

Location	**BACK g's**	**MIDDLE g's**	**FRONT g's**
A	________	________	________
B	________	________	________
1	________	________	________
2	________	________	________

PART 6 Using the velocity and acceleration for each dip, calculate the radius of curvature for the first 4 dips.

1 ______________________________ **2** ______________________________

3 ______________________________ **4** ______________________________

PART 7 What is the average velocity for the entire ride? ________

PART 8 Excluding the time it takes for the roller coaster to travel up the first hill, estimate the length of the remaining track, measure the time it takes for the train to travel this length and calculate the average velocity for this section of track.

Length: ________
Time: ________
Average Velocity: ________

PART 9 Estimate the height of the first DROP. ________

PART 10 Use your estimated numbers and any other numbers you need to measure and/or calculate to calculate the height of the first drop using conservation of energy methods.

Height from energy relationships: ______________________________

You are a secret agent from a competing amusement park. You task is to take a roller coaster and find out <u>EVERYTHING</u> you can about the roller coaster.

NEATNESS COUNTS!

YOUR ROLLER COASTER IS THE

____

Group Member's

____

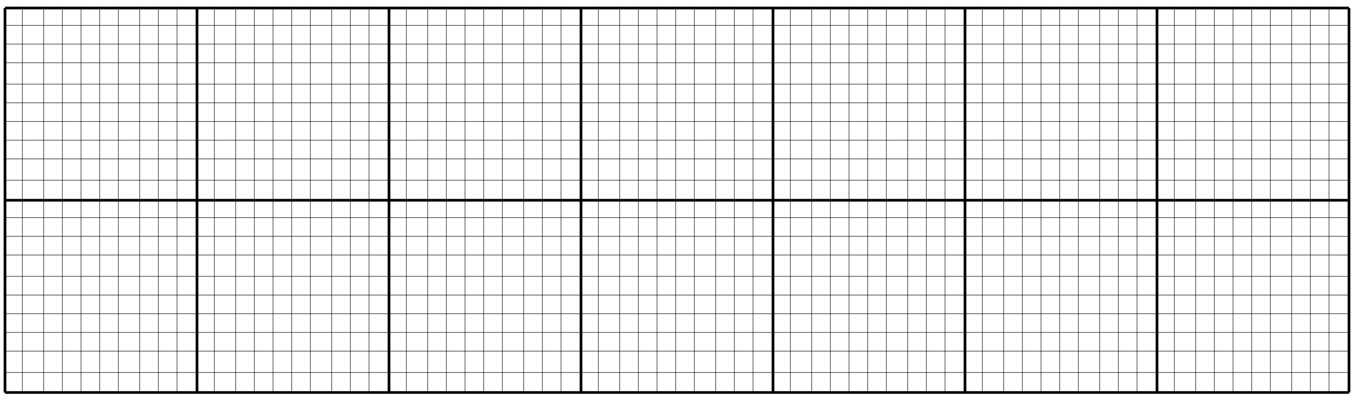
____

____

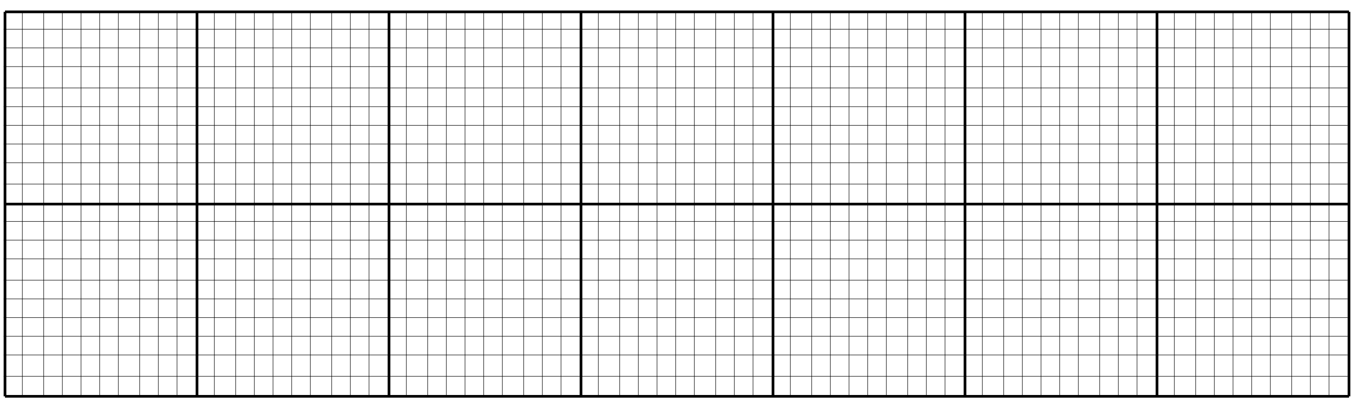
____

____

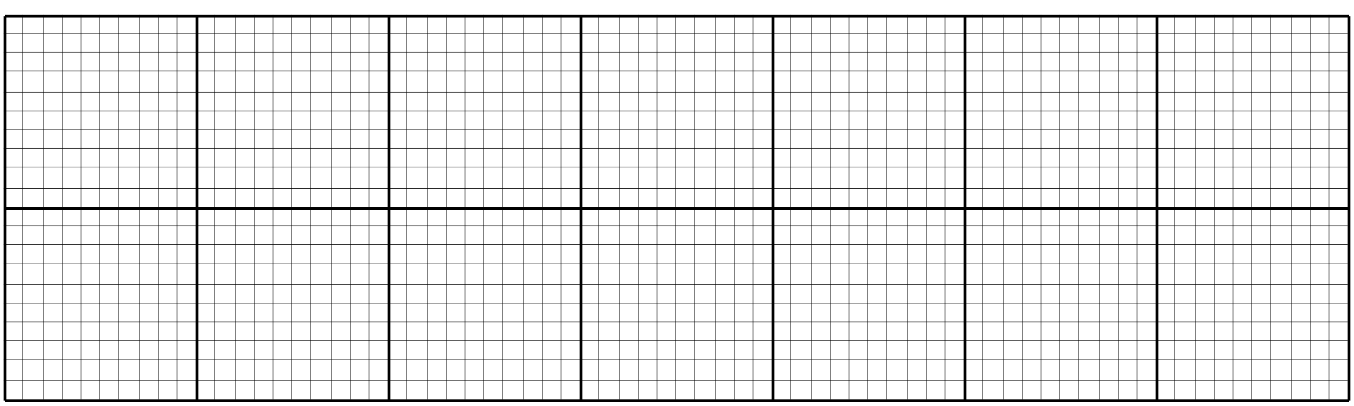
PART 1 Most roller coasters wrap themselves in a circle. Draw a **SCALE** drawing of your roller coaster's first **5 peaks and dips** if you could straighten it out. (You do not have to use all the horizontal length of this graphing space -the numbers are just for reference.)



START 1 2 3 4 5 6 7



8 9 10 11 12 13 14 15



16 17 18 19 20 21 22 23

- Label each **hill's peak as a letter, A - E** on your scale drawing. You do not have to use every letter. Note, the top of a loop counts as a hill peak.
- Label each **dip as a number, 1 - 5** on your scale drawing. You do not have to use every number. Note, each time the roller coaster is at the bottom of the loop counts as a different dip.

PART 2 Calculate the height at each peak.

PEAKS

A ________ **B** ________ **C** ________ **D** ________ **E** ________

PART 3 Calculate the velocity at each letter and number.

PEAKS

A ________ **B** ________

1 ________ **2** ________

	PACES	**METERS**
PART 4 How long is the length of the train of roller coaster cars.	________	________

How many cars make up the train? ________

How many rows of seats are in the train? ________

PART 5 Someone in your group needs to ride the roller coaster. Measure the g's at the first 4 dips and hills. Note which part of the car you are riding in at the time of the measurement.

DIP #	**g's**	**Number of rows of seats from the front** (middle is best)
1	________	________
2	________	________

PART 6 Measure the g's at the first 2 dips and the first 2 peaks after the initial hill from the back, middle and front of the train.

For each location indicate how many seats from the front you are riding.

Location	**BACK g's**	**MIDDLE g's**	**FRONT g's**
A	________	________	________
B	________	________	________
1	________	________	________
2	________	________	________

PART 7 What is the average velocity for the entire ride? ________

PART 8 Excluding the time it takes for the roller coaster to travel up the first hill, estimate the length of the remaining track, measure the time it takes for the train to travel this length and calculate the average velocity for this section of track.

Length: ________

Time: ________

Average Velocity: ________

PART 9 Which roller coaster(s) in the park has/have a hill that is NOT parabolic?

____

Which hill(s) was/were not parabolic? (1ST, 2ND, 3RD, ETC)

____

This page is to be used for your scratch work

Coaster Name ______________

Group Members

____

____

____

____

Length of the roller coaster train: ______________________________

- Label each hill or dip where a measurement was made.
- Record the time at each label on the sketch below - see the example sheet.
- Record the g's experienced at each hill or dip on the sketch below.
- Calculate the velocity at each labeled hill on the sketch below.
- Do <u>ALL</u> this for the first 4 hills and dips and loops. A loop counts as 1 hill and 2 dips. If your ride has a loop you need to do 5 dips.
- You need to also include horizontal distances between hills and dips.
- You need to neatly redraw the whole ride using straight edges and curves. The redrawn ride needs to be easy to see and clean. (Color coding helps.)

Use this page for your final design.

Roller Coaster name: ____________________

Group Members

____

____

____

____

- NEATNESS COUNTS A LOT.
- Draw it first with a pencil, then go over it with a felt tip for boldness. Color coding the drawing helps.
- Connect pages with tape if you need more room for your coaster drawing.
- Do not include support structures, only the curvature of the track.
- Make sure each hill and dip has time, velocity, g's felt, radius of curvature and height above the lowest point.
- You need to include horizontal distances between hills and dips.
- Accurately SCALE the vertical and horizontal parts of the graph paper.
- Each group member must turn in a complete drawing.

This page is to be used for your scratch work

Coaster Name *EXAMPLE*

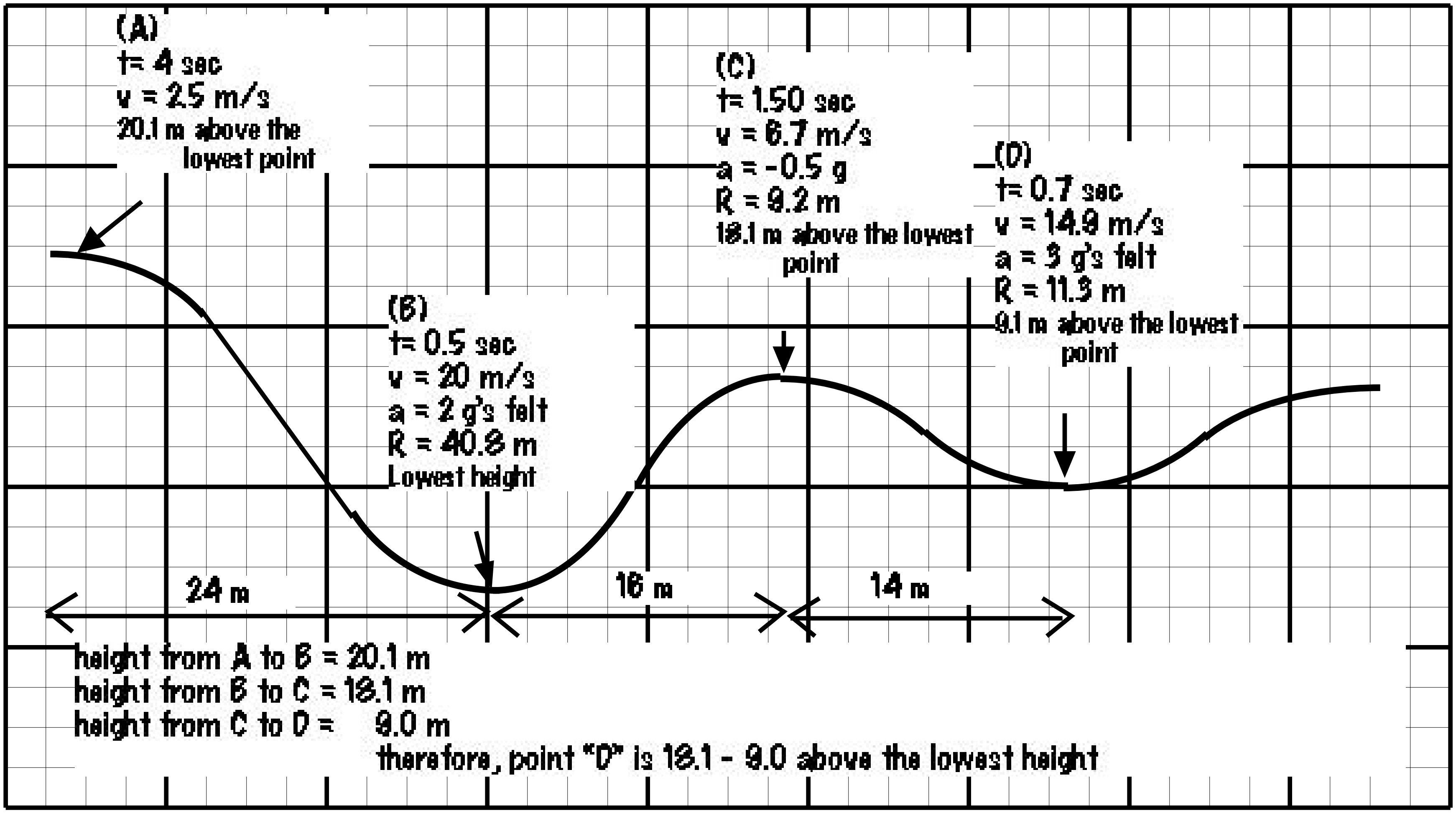
Length of the roller coaster train: *10 m*

Group Members
J. KIRK
J. Picard
B. Cisco
K. Janeway & T. Wayne

- Label each hill or dip where a measurement was made.
- Record the time at each label on the sketch below.
- Record the g's experienced at each hill or dip on the sketch below.
- Calculate the velocity at each labeled hill on the sketch below.
- Do ALL this for the first 4 hills and dips and loops.
- Estimate distances between hills and dips.

ROUGH DRAFT

The final draft would be scaled with all numbers neatly and boldly drawn.



The next lab was designed for the park that does not allow accelerometers to be taken on the rides themselves.

Assigned Ride		**Group Members**
☐ ____________	☐ ____________	1 ____________
☐ ____________	☐ ____________	2 ____________
☐ ____________	☐ ____________	3 ____________
☐ ____________		4 ____________

- **Neatness counts! If your work can not be clearly read and understood then it is of no value. Be meticulous.**
- **Show your work and make comments as to what the distance and time physically measures.**

EXAMPLE

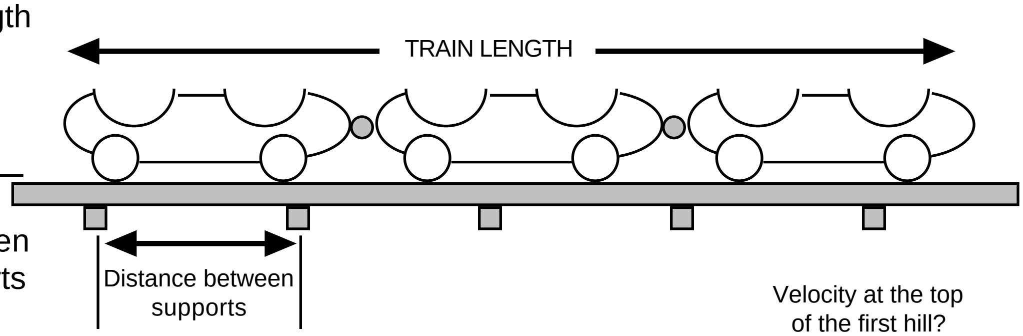
$$\text{velocity} = \frac{\text{train length}}{\text{time for train to pass the center track support at the bottom of the hill}} = \frac{8.5\ \text{m}}{0.85\ \text{s}} = 10\ \text{m/s}$$

- **You may get help from your notes, your text book, handouts or any other group members.**
- **YOU MAY NOT GET HELP FROM ANOTHER TEACHER.**
- **This is due the Wednesday after the trip.**

---

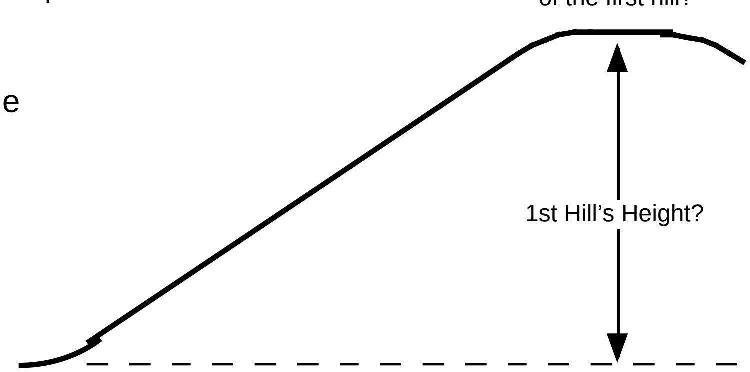
1 What is the length of the roller coaster train?

____



2 What is the distance between cross tie supports on the ride?

____



3 What is the published length of the track?

____

4 How many roller coaster cars make up a train of cars?

____

5 What is the mass of each coaster car?

____

6 What is the vertical height of the 1st hill that the roller coaster car is lifted up?

____

7 How fast is the car traveling over the top of the hill?

____

8 How much time does it take for the middle of the train to reach the highest point of the first hill?

____

9 What is the velocity of the train at the bottom of the dip after the first hill?

____

10 Using the velocity in the previous question calculate the height of this drop.

____

If the coaster has a loop, answer questions 11-16.

11 What is the velocity of the roller coaster at the bottom of the 1st loop?

____

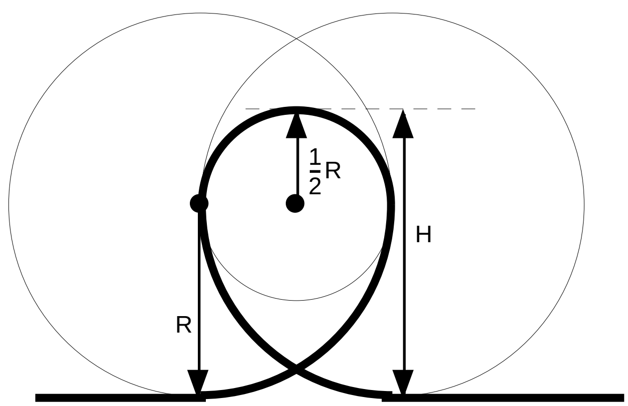
12 What is the velocity of the roller coaster at the top of the loop?

____

13 Using the information from the previous questions, calculate the vertical height of the loop.

____

14 Suppose your loop was designed using simple geometry. Below is an irregular loop shape that is designed from the splicing together of two large circles with a smaller circle. The smaller circle's radius is half the larger circle's radius. Using the information from the previous problem calculate the radii of the two circles.



Bigger circle's radius ______________

Smaller circle's radius ______________

15 Using the calculated radius information above, calculate the g's felt by a rider at the bottom of the loop and at the top of the loop.

____

16 Using your previous answers do you think it is possible that the loop is designed according to our simple geometry model?

____

17 Calculate the velocity of the car as it travels over the 2nd hill.

____

18 Calculate the velocity of the car as it travels past the bottom of the next dip.

____

19 Calculate the velocity of the car as it travels over the 3rd hill.

____

20 Calculate the velocity of the car as it travels past the bottom of the next dip.

____

21 Assuming the initial total mechanical energy of the train is ZERO, how much total mechanical energy is gained by raising the train of cars to the top of the first hill.

____

22 The train has to lose its total mechanical energy by the time it reaches the end of the ride. Assuming 2/3's of this initial energy at the top of the hill is lost due to friction during the length of the entire ride, what is the average force of friction opposing the train's motion as it travels along the entire length of the track? (HINT: Use energy and work)

____

23 How much horsepower was used to raise the train up the first hill?

____

24 If electricity costs \$0.20/(kW•hr), then how much does it cost to raise the train up the first hill?

____

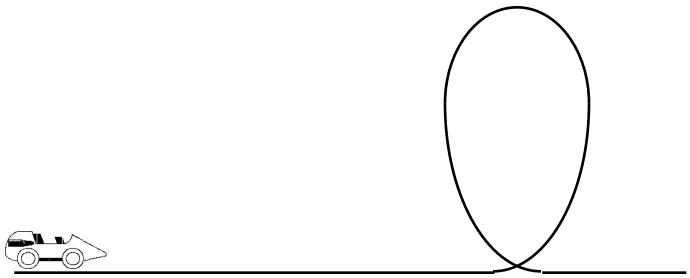
25 How many runs does a train make in 1 hour?

____

26 How much does it cost to run the train in a 14 hour day?

____

The "Absentee Roller Coaster Lab" is meant to be substitute lab for those students not going to the park.



Your Name: ____________________
____

Your Grader: ____________________
____

Due Dates:

- ____________ : Turn in your roller coaster with your answers.
- ____________ : Turn in your roller coaster with your answers and the other groups answers to your roller coaster -No late papers accepted. This counts as a lab grade.

- You may work in groups of 2.

Your roller coaster must be neatly drawn in pen. It must take up no more than one 8.5 X 11 inch piece of paper.
Your questions are to be written on a separate sheet of paper.
Your solutions are to be on a third sheet -SHOW ALL WORK OR LOSE 8 POINTS.
Do not give your answers to the group checking your work. After they have finished checking your work compare answers and solve any discrepancies among yourselves.

Your (group's) roller coaster's points depend on if your roller coaster includes:

__ Work calculated from Fd equation -with at least 1 question about it. (7 pts.)

__ Work calculated from a graph of force vs. distance-with at least 1 question about it. The graph cannot be a horizontal line. (7 pts.)

__ A loop-the-loop -with at least 2 questions about it. (7 pts.)

__ A spring -with at least 1 question about it. (7 pts.)

__ A hill -with at least 2 question about it. (7 pts.)

__ At least one of the above questions must be about the g's felt by the rider. (7 pts.)

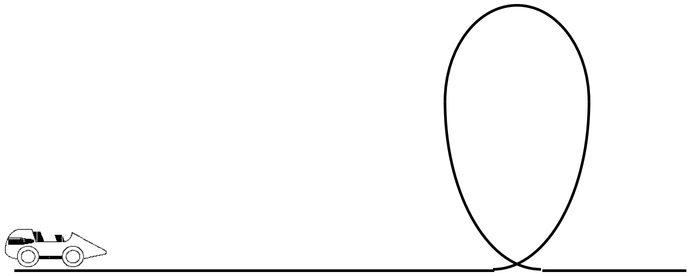
__ The rider experiences g's no higher than 10 g's at every hill top and dip. (7 pts.)

__ Calculated the solutions to someone else's lab. (25% of the lab grade.)
Whose lab did you grade?

__ Percentage of your lab's questions that were correct as graded by another group.

__________ (20 pts. maximum)

Total number of points checked above: ☐



Your Name: ______________________

____

Your Grader: ______________________

____

Overview:

For this project, you will create a poster that diagrams a made up roller coaster. This roller coaster will contain calculations of g's, velocities, heights, spring constants, etc.

Due Date __________

- No late papers accepted.
- This counts as a lab grade.
- You may work in groups of 2 or by yourself.

Presentation guidelines:

_ Neatly drawn with a magic marker.
straight lines drawn with a straight edge or computer
smooth curves drawn with drawing aides or a computer
no white out
virtually no smudges
grid lines drawn every set distance with a ball point pen

_ Size
bigger than 10" X 13"
smaller than 30" X 36"

_ Must be drawn on poster board or mounted to poster board

_ The board must have the coaster without any numbers drawn on it.

_ The board must have a plastic sheet cover.

- The cover contains all numbers and answers. These numbers are to be placed at the corresponding locations on the coaster track.
- The plastic cover must be attached only at the top of the poster board.

Calculation guidelines:

_ Every hill, dip and loop, top and bottom.
Calculate and label the velocity, centripetal acceleration expressed in m/s^2, and the g's felt by the rider.

_ Every linear force (either from a graph or pure number)
Calculate and label the force at the beginning where the force is applied and at the end of where the force is applied. Also calculate the acceleration due to this force expressed in m/s^2 and g's.

_ Every spring
Calculate and label the spring constant.
Calculate and label maximum compression distance of the spring.
Calculate and label the velocity of the coaster's car before it hits the spring.

_ EVERY SINGLE CALCULATION must be NEATLY written on a 1/2 sheet of paper.

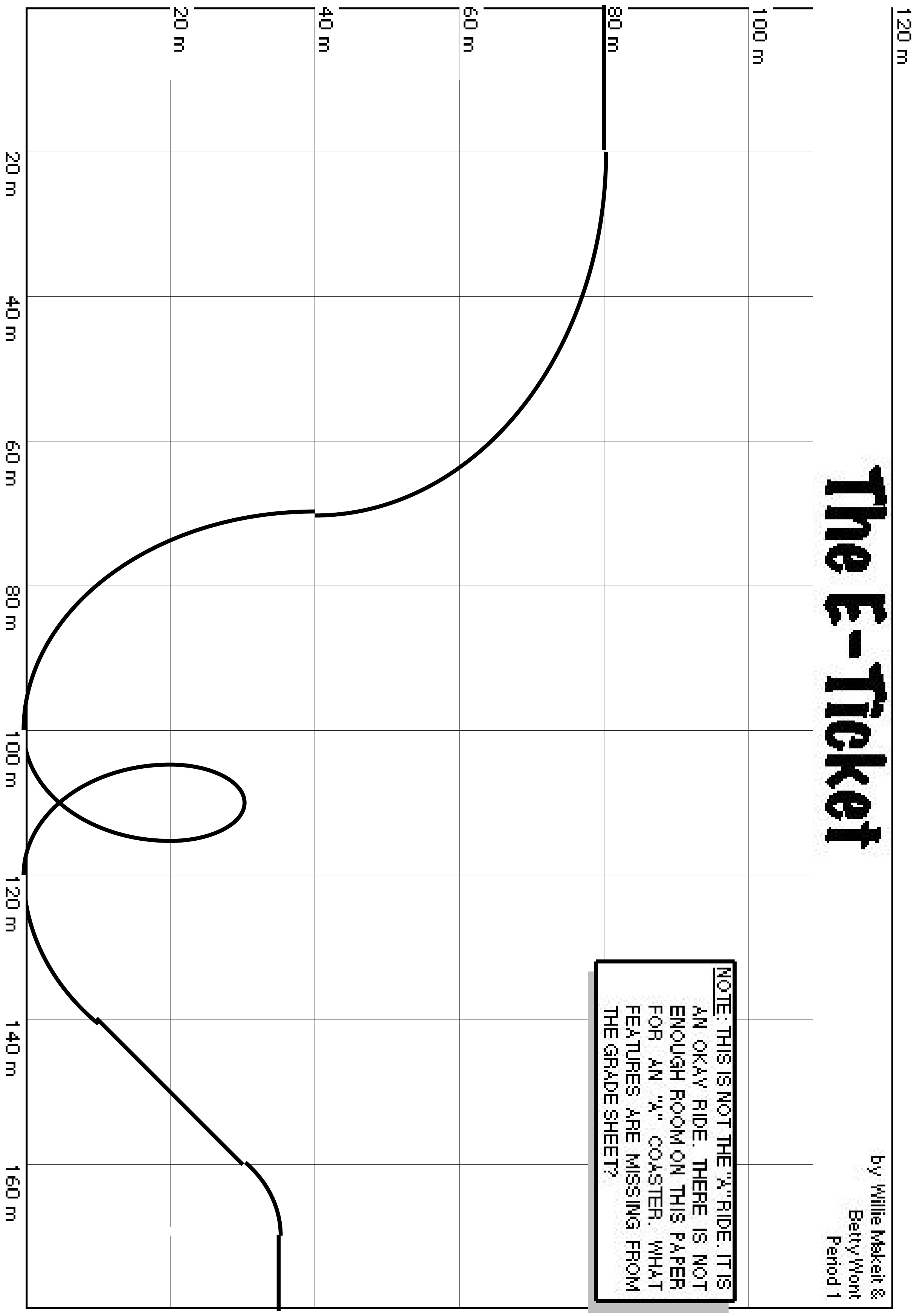
Other

- _ Maximum positive g's must be less than 10 g's to be considered for an "A" grade.
- _ Maximum negative g's must be less than 4 g's to be considered for an "A" grade.
- _ Must contain at least one loop
- _ Must contain one spring <u>OR</u> force that does work -whose force is calculated from a graph.

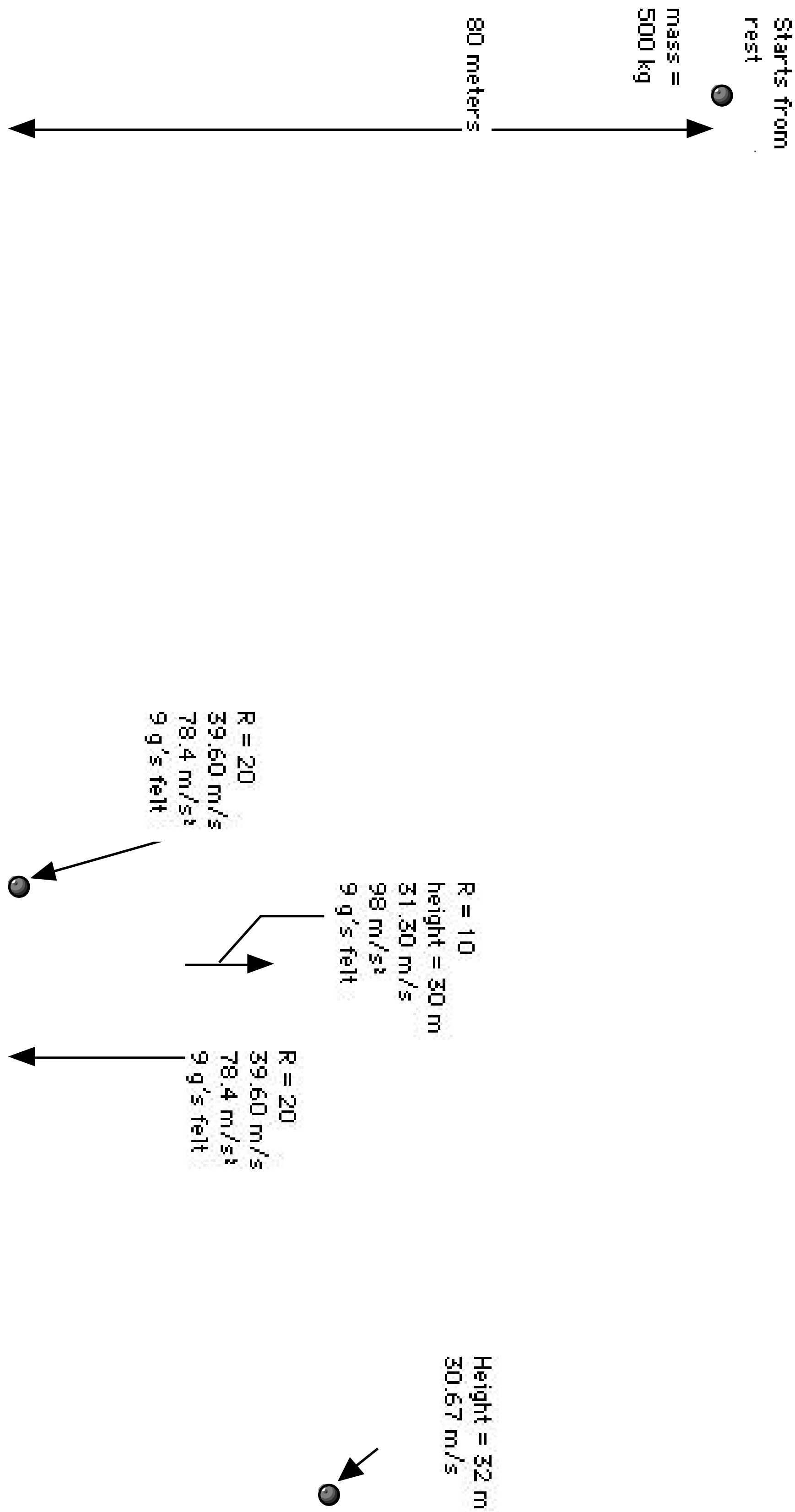
GRADES

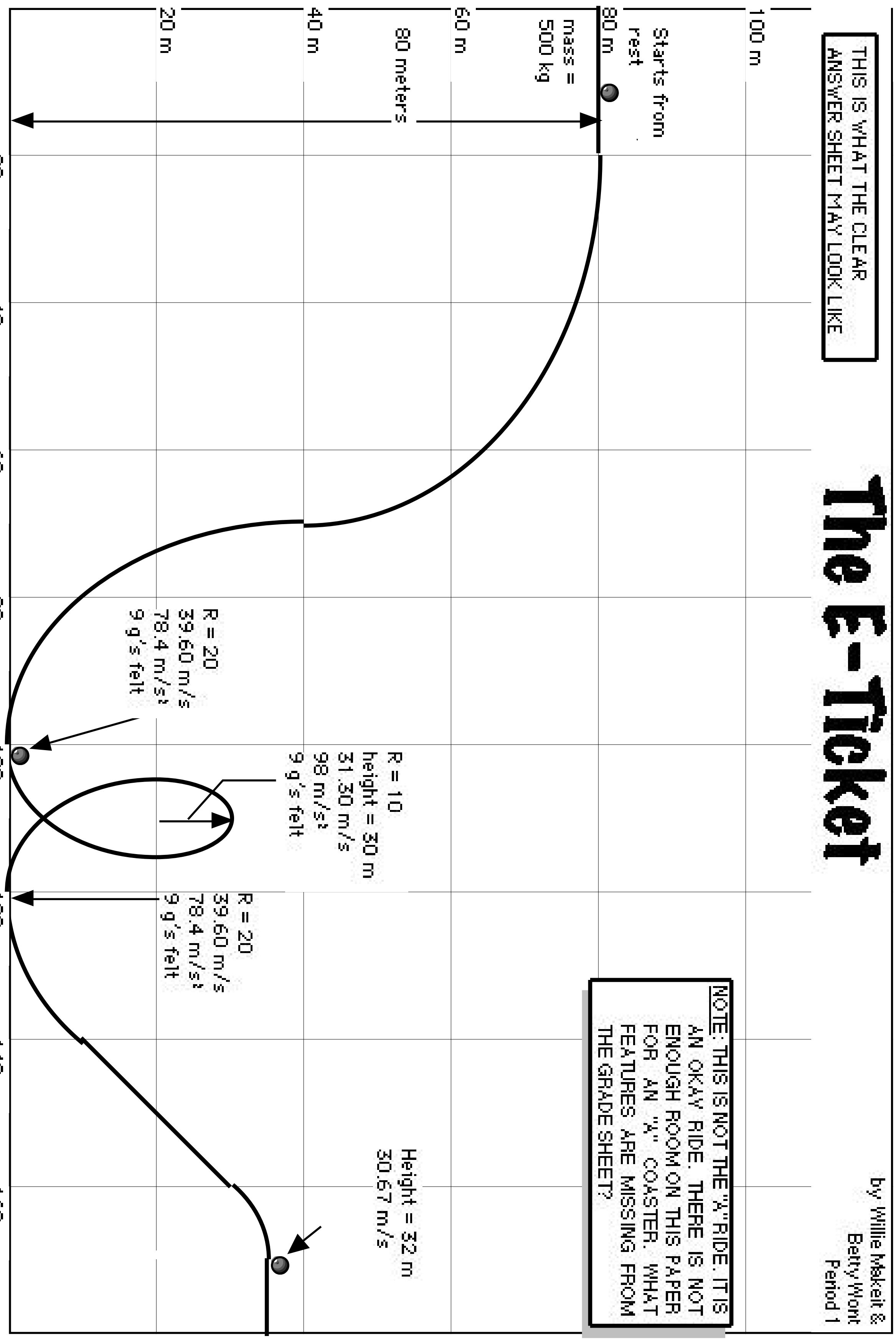
Grades are based on the neatness of your presentation and the correctness of your work. It is hard to get an "A." An "A" project follows all the rules above and is creative. It is the exemplary project. It is easier to get a "B."

Partial Example



THIS IS WHAT THE CLEAR ANSWER SHEET MAY LOOK LIKE

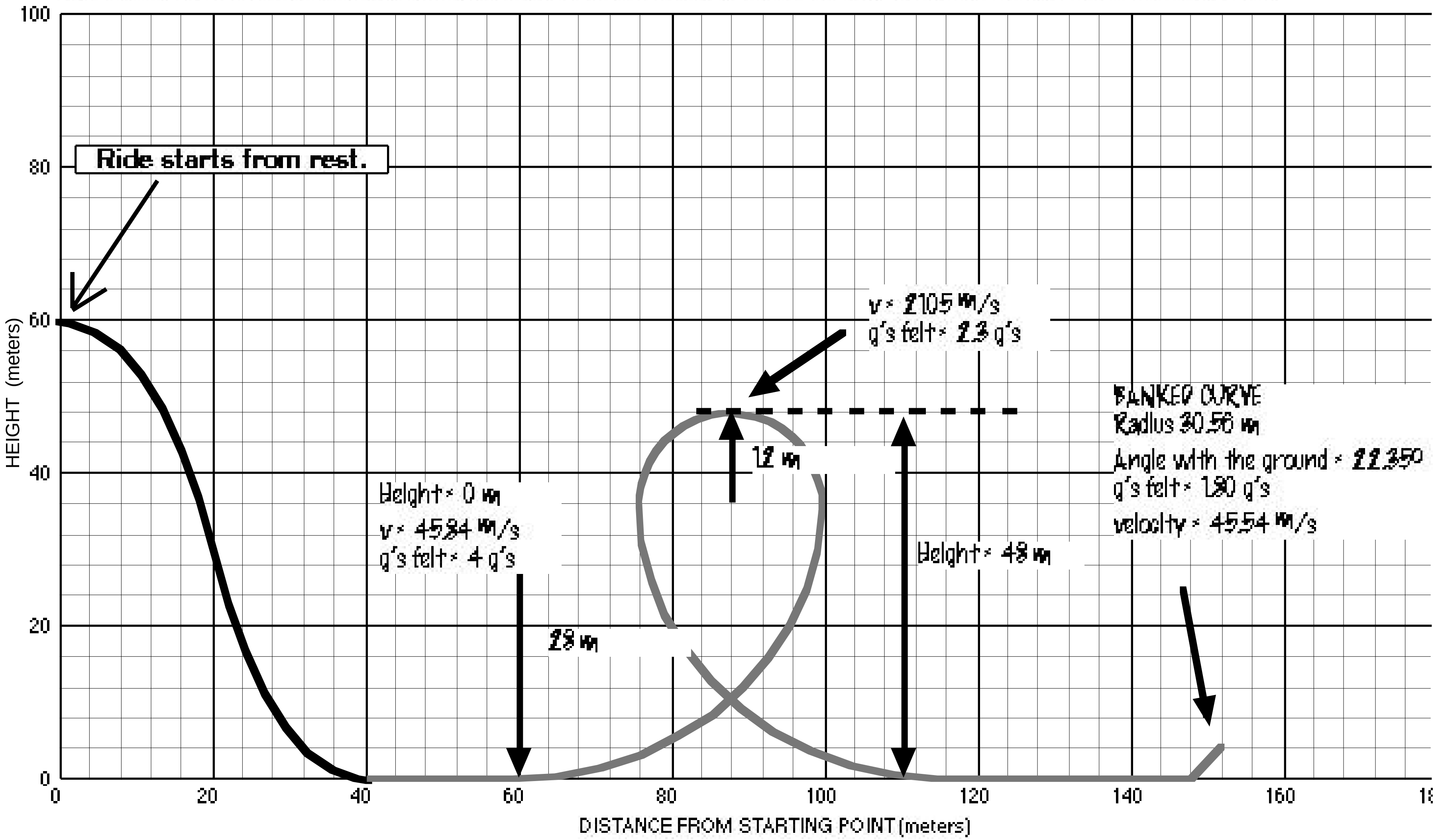



The E-Ticket
by Willie Makeit &
Betty Wont
Period 1
THIS IS WHAT THE CLEAR ANSWER SHEET MAY LOOK LIKE
120 m
100 m
80 m
60 m
40 m
20 m
Starts from rest
mass = 500 kg
80 meters
R = 20
39.60 m/s
78.4 m/s²
9 g's felt
R = 10
height = 30 m
31.30 m/s
98 m/s²
9 g's felt
R = 20
39.60 m/s
78.4 m/s²
9 g's felt
Height = 32 m
30.67 m/s
NOTE: THIS IS NOT THE "A" RIDE. IT IS AN OKAY RIDE. THERE IS NOT ENOUGH ROOM ON THIS PAPER FOR AN "A" COASTER. WHAT FEATURES ARE MISSING FROM THE GRADE SHEET?
20 m
40 m
60 m
80 m
100 m
120 m
140 m
160 m

Design a roller coaster according to the following criteria: Name Example

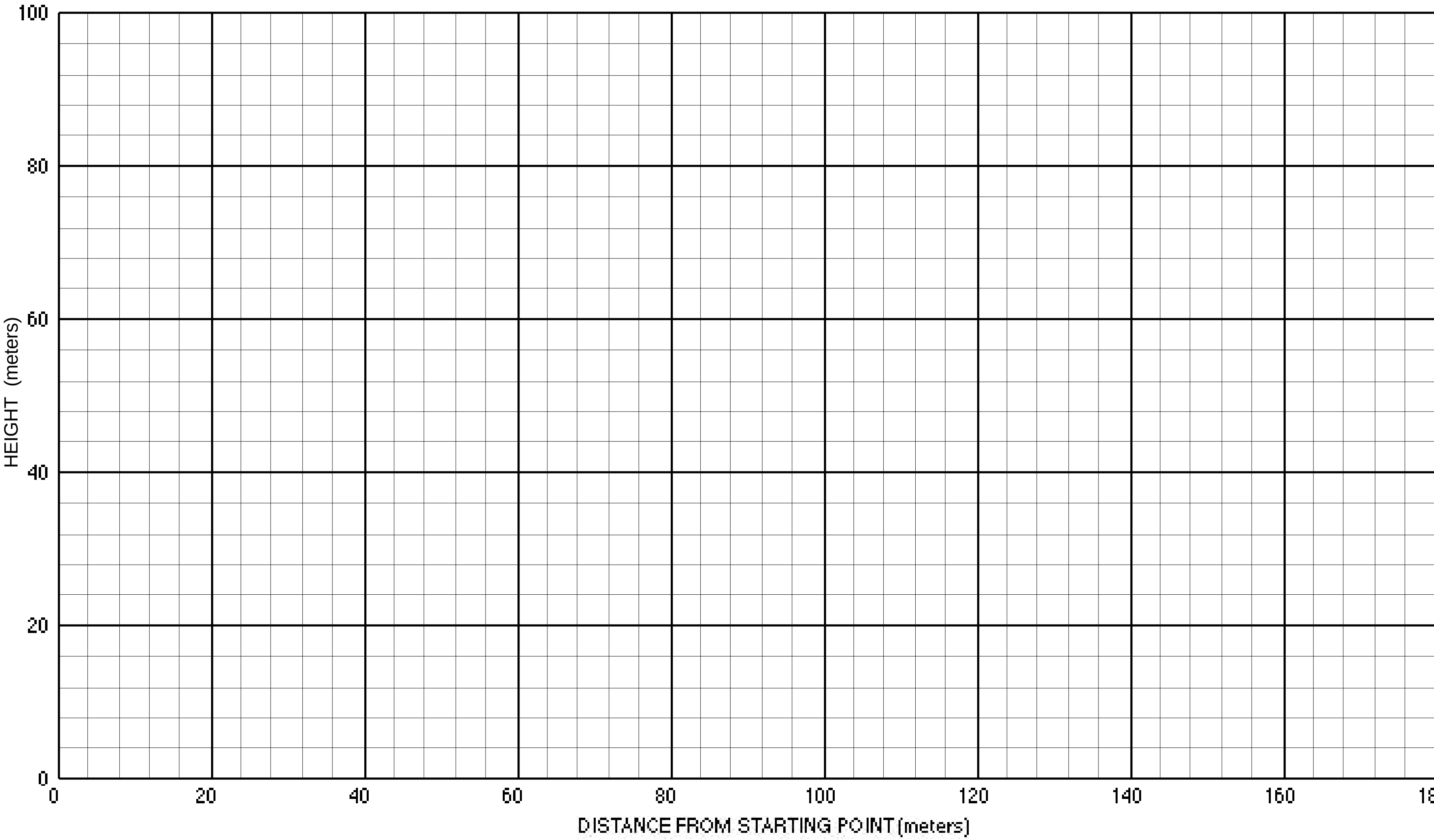
- The g's felt by the rider as the coaster enters any loop are to be less than 5 g's.
- The coaster car is not to fall off the top of the loop.
- The loop can be a regular circular loop or an irregular shaped loop.
- After the loop there is to be a curve. You are not expected to draw this element to scale. The rider is to experience less than 2 g's around the loop.
- The drawing is to be to scale. Everything is to be proportional. Everything is to be NEAT -Straight lines. No smears. All numbers are shown of the diagram.
- Use 2 distinct colors, NOT black and blue. Use black and green, blue and red, black and red or blue and green. Use one color for the design and one color for the numbers.

Note: This is an example of the quality of the work to be turned in. The example numbers shown below are made up. Do not try to duplicate them with math or physics. You are to show at least this amount of information.



Design a roller coaster according to the following criteria: Name ____________________

- The g's felt by the rider as the coaster <u>enters</u> any loop are to be <u>less than</u> 5 g's.
- The coaster car is not to fall off the top of the loop.
- The loop can be a regular circular loop or an irregular shaped loop.
- After the loop there is to be a curve. You are not expected to draw this element to scale. The rider is to experience <u>less than</u> 2 g's around the loop.
- The drawing is to be to scale. Everything is to be proportional. Everything is to be NEAT -Straight lines. No smears. All numbers are shown of the diagram.
- Use 2 distinct colors, NOT black and blue. Use black and green, blue and red, black and red or blue and green. Use one color for the design and one color for the numbers
- You are not limited to the number of hills, dips, loops or other imaginable elements. Your ride must include at least one loop and at least 1 banked curve.
- Your lab <u>WILL BE</u> compared to other group's designs for a grade. It is due the Monday after receiving this sheet. You may connect sheets to make a larger sheet.

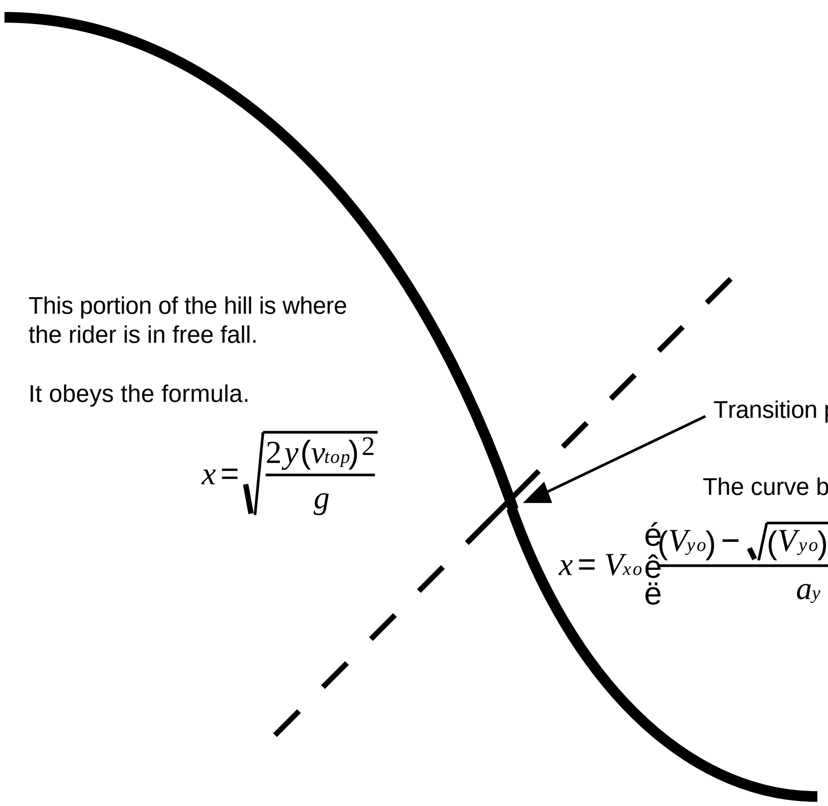


HOW TO DESIGN YOUR OWN ROLLER COASTER

Roller coasters are about speed control and sensations. When designing a roller coaster the designer needs to be able to predict speeds and the riders' sensations.

THE FREE-FALL DROP

A free fall hill is defined by two formulae. The first formula is for the top of the hill. The second formula is for the bottom of the hill. The ending and beginning locations for the top and the bottom of the hill meet at the transition point. The transition point is the location of the maximum angle. The designer usually defines this angle somewhere between 35° and 60°.



This portion of the hill is where the rider is in free fall.

It obeys the formula.

$$x = \sqrt{\frac{2y(v_{top})^2}{g}}$$

Transition point where the hill is at its maximum angle

The curve below the transition point obeys the formula.

$$x = V_{xo}\left[\frac{(V_{yo}) - \sqrt{(V_{yo})^2 - 2(a_y)y}}{a_y}\right] - \frac{a_x}{2}\left[\frac{(V_{yo}) - \sqrt{(V_{yo})^2 - 2(a_y)y}}{a_y}\right]^2$$

The equation for the section of the hill before the transition point is

$$y = \frac{gx^2}{2v^2}$$

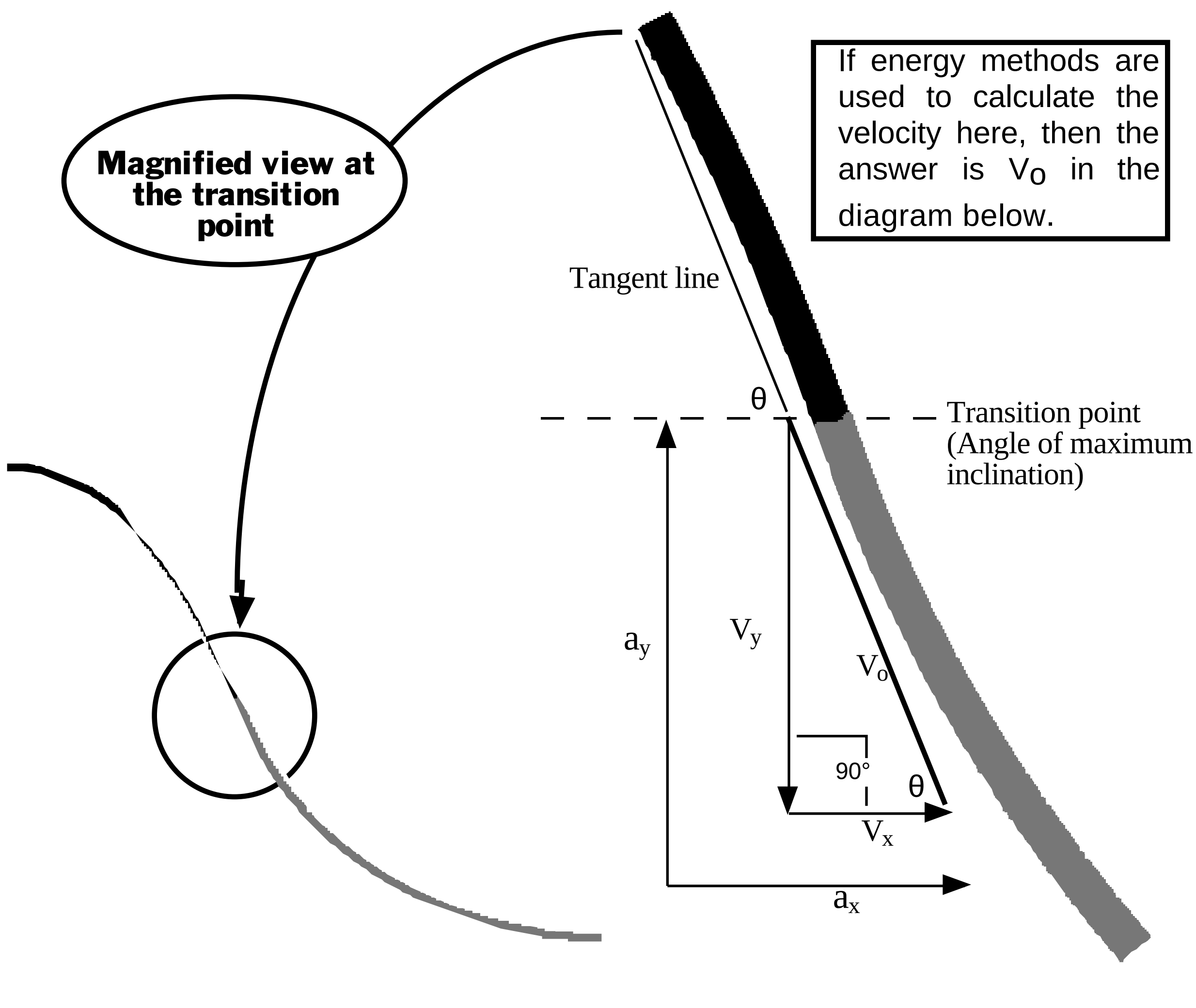
y = the height from the top of the hill
x = is the distance away from the center of the hill
v = the velocity the roller coaster car travels over the top of the hill.
a = the acceleration due to gravity. 9.80 m/s^2 for answers in meters. 32.15 ft/s^2 for answers in feet.

There is a point on the hill where it no longer follows the equations of projectile motion. This is the transition point. The transition point is the location on the hill where its angle is at a maximum. Below the transition point the curve is created following some different rules.

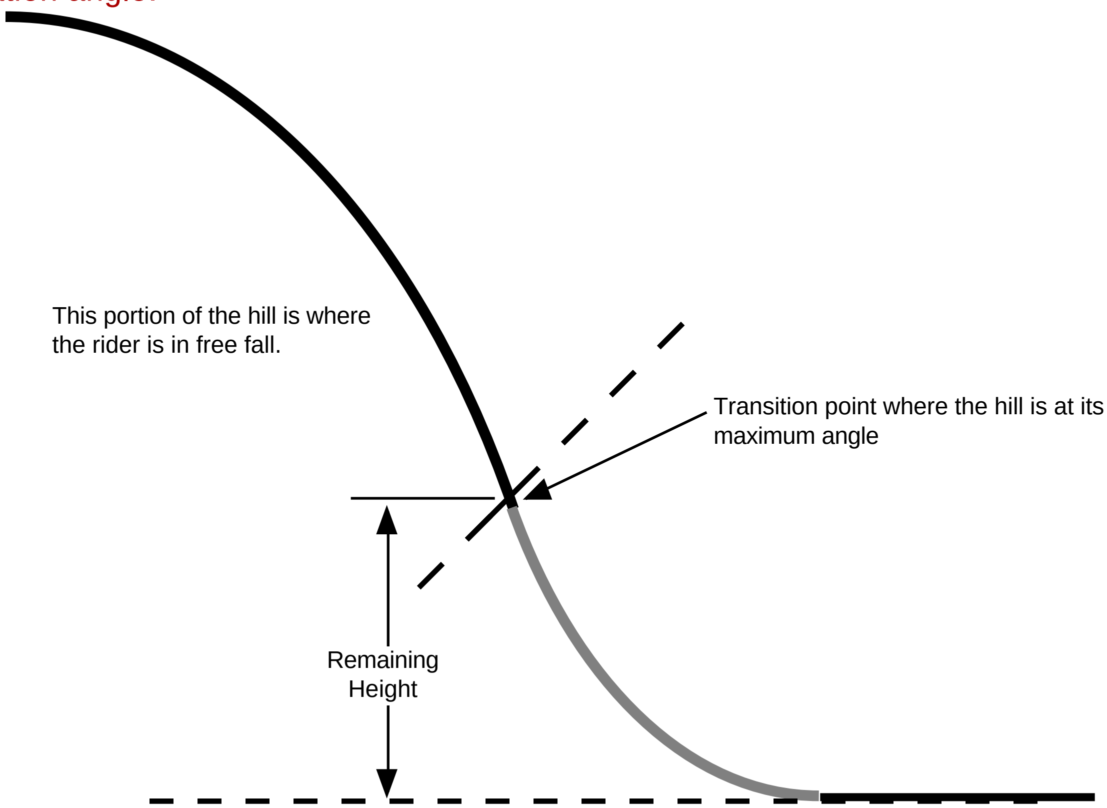
1) To design a hill from top to bottom decide on the initial velocity as the car travels over the hill, the height of the hill, and the maximum inclination angle. The inclination angle will determine how steep the hill will become. Choose a value between 35° and 60° as a "ball park" starting figure. The bigger the maximum inclination angle, the greater the acceleration at the bottom on a given hill. If the acceleration at the bottom of the hill is too great, make the hill higher or reduce the maximum inclination angle. As the free-fall part of the curve is plotted keep track of the angle between successive points. When the angle of the hill matches or just exceeds the maximum angle of inclination of the hill, stop using the free-fall equations.

   When doing the actual calculations, use small vertical step intervals, e.g. 1.0 meter increments. Make the top of the hill zero. For each y step down you will get an x value. These are your x, y coordinates for the hill's shape.

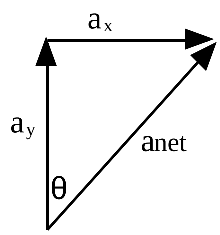
2) Calculate the velocity of the bottom of the hill using energy relationships. $[(v_{TOP})^2 = (v_{BOTTOM}^2) + 2gh]$



3) Calculate the remaining distance to the bottom of the hill from the location of the maximum inclination angle.



4) Decide on an acceleration that will be used to change the shape of the hill. Say 1.5 to 2.5 g's (14.7 to 24.5 m/s2). There are two accelerations the coaster car will feel. One vertically will slow its vertical velocity to zero. A second horizontal velocity will increase the car's velocity to the final velocity. The total acceleration is



$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

$$a_{net} = \sqrt{(a_x)^2 + (a_y)^2}$$

Remember,

g's felt = ay + 1g

in the vertical direction. In the horizontal direction g's felt = a_x. To calculate the g's felt by the rider use one of the g's felt values when calculating a_{net}.

5) Calculate the vertical component of velocity at the location of maximum inclination angle. Calculate the horizontal component of velocity at the location of maximum inclination angle. These velocities are the initial velocities for the final section of the track.

6) Use the formula

$$x = V_{xo}\left[\frac{(V_{yo}) - \sqrt{(V_{yo})^2 - 2(a_y)y}}{a_y}\right] - \frac{a_x}{2}\left[\frac{(V_{yo}) - \sqrt{(V_{yo})^2 - 2(a_y)y}}{a_y}\right]^2$$

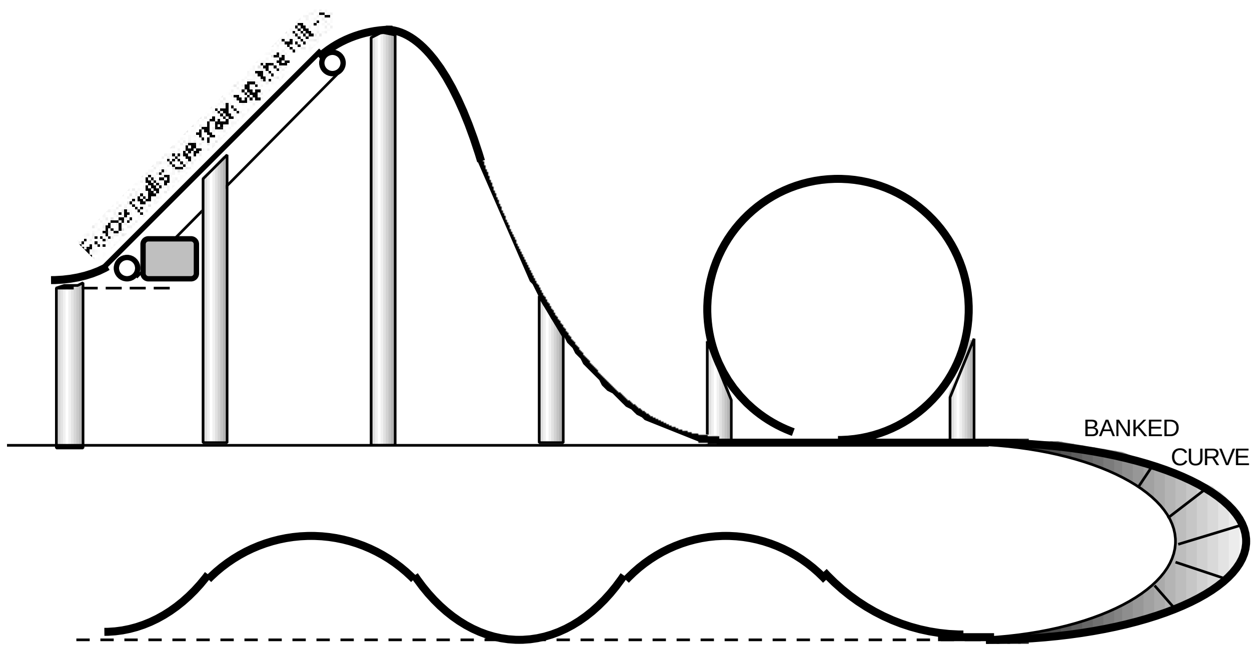
to calculate the bottom half of the hill.

Introduction to Roller Coaster Design (Example)

Intro to Design Example

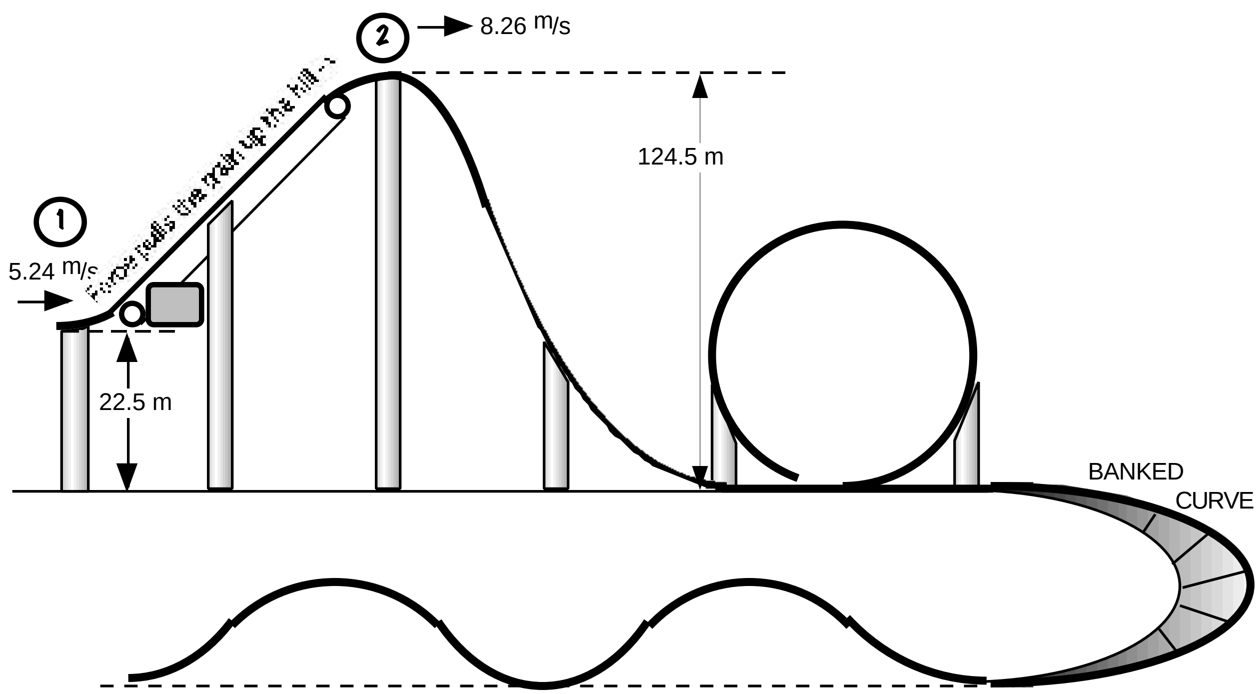
This will show the reader the basic steps to designing a roller coaster. The example coaster will not be the best possible design. (I don't want students to use it as their own in other projects.)

STEP 1 Draw a picture of what the coaster *may* look like.



STEP 2 Assign some beginning numbers.

Numbers like the initial velocity as the coaster train leaves the station. The mass of the coaster train. And/or an initial velocity as it tops the first hill. (Label the pieces for easy identification when analyzing.)

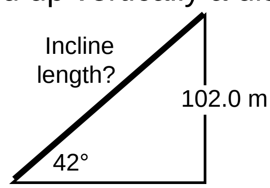


STEP 3 Begin to calculate everything you can and check to see if it makes sense.

For this design start by calculating the force needed to pull the train up the incline and the power to pull it up the incline.

In order to calculate the power force and power to pull it up the incline I'm going to need the train's mass. So make up a reasonable mass. This coaster is made up of 6 cars. Each car has a maximum mass, with two riders at 100 kg each, of 735 kg; (535 kg car + 100 kg rider + 100 kg rider.) Therefore, the coaster train will have a mass of 4410 kg.

What angle will the first incline be? The designer can choose this number too. 42° is good. The train is going to be pulled up vertically a distance of, (124.5-22.5), 102.0 m.



$$\text{Incline length} = {}^{102}/_{(\sin 42°)}$$
$$\text{Incline length} = 152.4 \text{ m}$$

$$ET_{(\text{OUT OF STATION})} + \text{Work} = ET_{(\text{TOP OF }1^{st}\text{ HILL})}$$
$$KE + PE + W = KE + PE$$
$$(1/2)mv^2 + mgh + Fd = (1/2)mv^2 + mgh$$
$$(1/2)4410(5.24)^2 + 4410(9.8)(22.5) + F(152.4) = (1/2)4410(8.26)^2 + 4410(9.8)(124.5)$$
$$60544.008 + 972405 + F(152.4) = 150441.858 + 5380641$$
$$F(152.4) = 4498133.85$$
$$\underline{F = 29515.314 \text{ N}}$$ ... is the pulling force along the incline.

How much time will it take to travel up the incline?

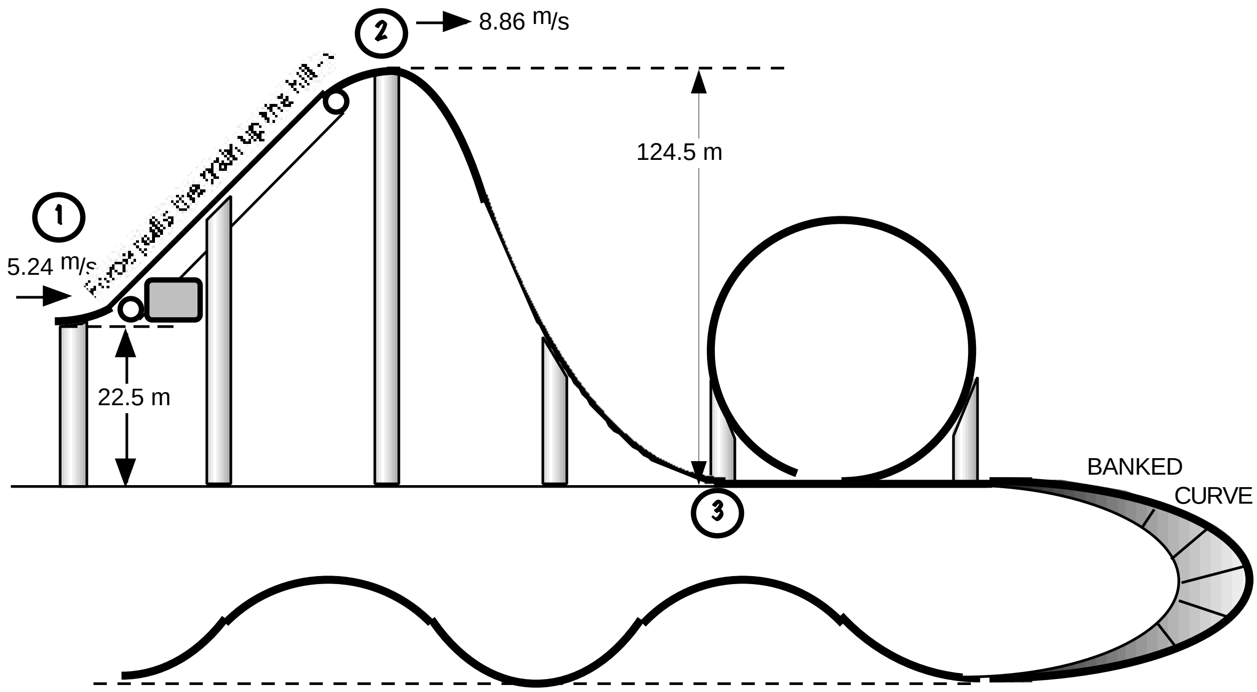
The acceleration of the train is found from

$$(v_f)^2 = (v_o)^2 + 2ad$$
$$(8.26)^2 = (5.24)^2 + 2(a)d$$
$$a = 0.134 \text{ m/s}^2$$

$$v_f = v_o + at$$
$$8.26 = 5.24 + 0.134(t)$$
$$\underline{t = 22.537 \text{ sec}}$$ ...is the time to climb the incline. (Most initial lift times are between 60 and 120 seconds.)

It is beyond the scope of this book to show how to calculate the time for each track element. It was shown here because of its ease of calculation.

STEP 4 Calculate maximum velocity of the ride.



Since the 1st drop is the longest, the velocity at the bottom will be the greatest, (location #3). Energy relationships will be used to calculate the velocity.

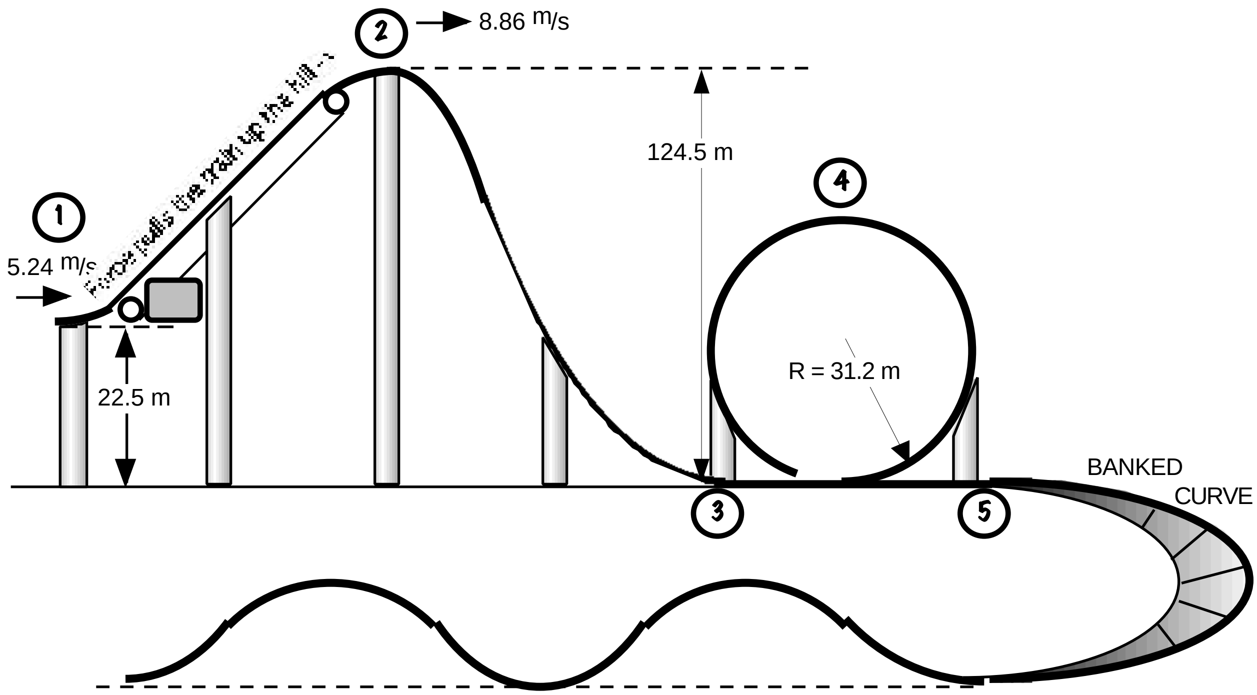
$$ET_{(LOCATION\ \#2)} = ET_{(LOCATION\ \#3)}$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$
$$(1/2)4410(8.26)^2 + 4410(9.8)(124.5) = (1/2)4410(v)^2 + 4410(9.8)(0)$$
$$150441.858 + 5380641 = 2205(v)^2$$
$$2508.428 = (v)^2$$
$$v = 50.084$$

$\underline{v = 50.1}$ m/s ... At the bottom of the first hill

That's 112 mi/hr !!!

STEP 5 The loop.

For any loop, the designer would like to know the velocity as the rider enters the loop; at the top of the loop; and as the rider leaves the loop. The designer would also like to know the g's felt by the passengers. This is the location on the ride where riders are most likely to pass out if the g's are too much. The radius in the loop below is made up.



The velocity as the rider enters the loop and as the rider leaves the loop is the same as the velocity at the bottom of the first hill. This is because all three locations are at the same height.

The velocity at the top of the loop is not the same as at the bottom. As the coaster travels up the loop it will lose kinetic energy and gain potential energy.

The height of the loop is simply double the radius. h = 2(31.2) = 62.4 m

$$ET_{(LOCATION\ \#2)} = ET_{(LOCATION\ \#4)}$$

$$KE + PE = KE + PE$$

$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$

$$(1/2)4410(8.26)^2 + 4410(9.8)(124.5) = (1/2)4410(v)^2 + 4410(9.8)(62.4)$$

$$150441.858 + 5380641 = 2205(v)^2 + 2696803.2$$

$$1285.388 = (v)^2$$

$$v = 35.852$$

$\underline{v = 35.9}$ m/s ... At the bottom of the first hill

That's 80.3 mi/hr !!!

If you are doing this calculation and you get an expression that requires you to calculate the velocity by taking the SQUARE ROOT OF A NEGATIVE NUMBER, then the loop is too tall for the given velocity at the bottom of the loop. The velocity will need to be increased or the height of the loop decreased.

To calculate the g's felt by the rider, calculate the centripetal acceleration at each location, convert to g's and either add or subtract a g as necessary.

As the rider enters and leaves the loop

v = 50.084 m/s

r = 31.2 m

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{50.084^2\ \mathrm{m/s}}{31.2\ \mathrm{m}}$$

$$a_c = 80.398\ \mathrm{m/s^2}$$

$$a_c = \frac{80.398\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}}$$

a_c = 8.2 g’s

a_c = 8.2 g’s + 1g

a_c = 9.2 g’s ... That is an incredible amount of g’s. Most coasters do not go above 5 g’s. To be safe the radius at the bottom of the loop needs to be bigger.

As the rider passes the top of the loop

v = 35.852 m/s

r = 31.2 m

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{35.852^2\ \mathrm{m/s}}{31.2\ \mathrm{m}}$$

$$a_c = 41.198\ \mathrm{m/s^2}$$

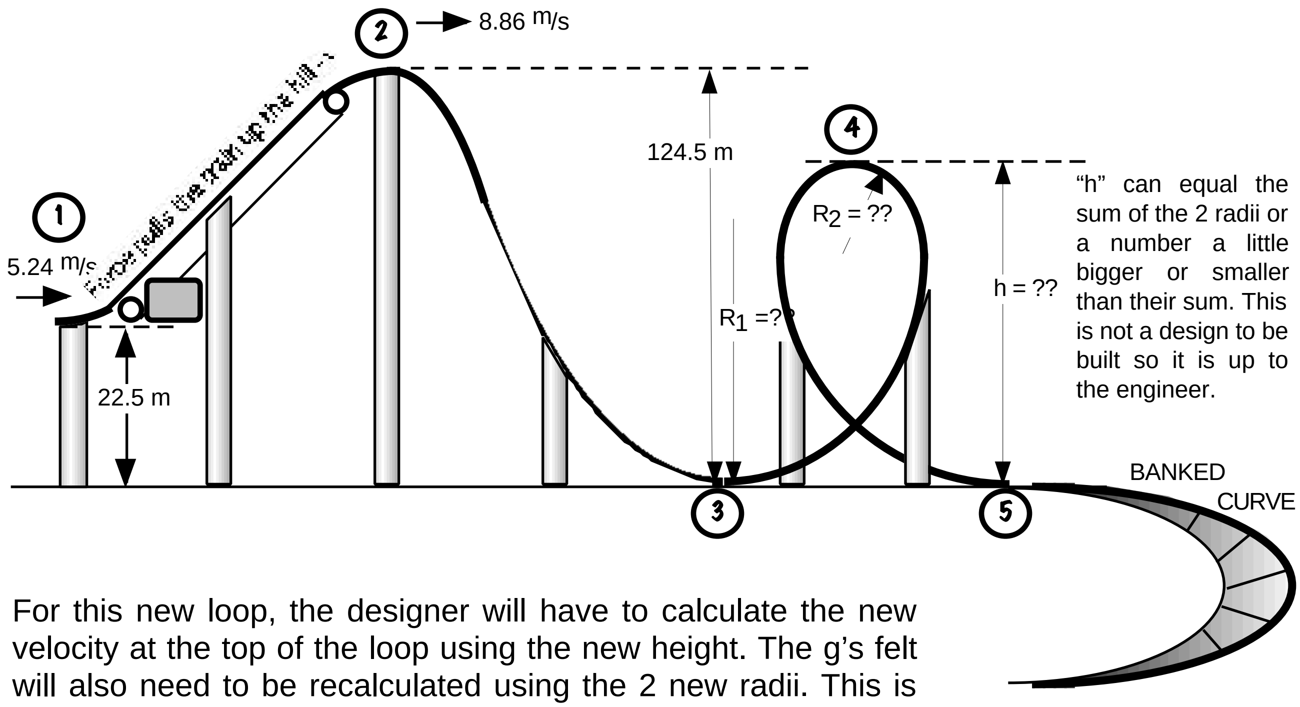
$$a_c = \frac{41.198\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}}$$

a_c = 4.2 g’s

a_c = 4.2 g’s - 1g

a_c = 3.2 g’s ... That is an acceptable amount. But 3.2 g’s is rather high for the top of a loop. Most of the time the g’s at the top of a loop are from 1.5 to 2 g’s.

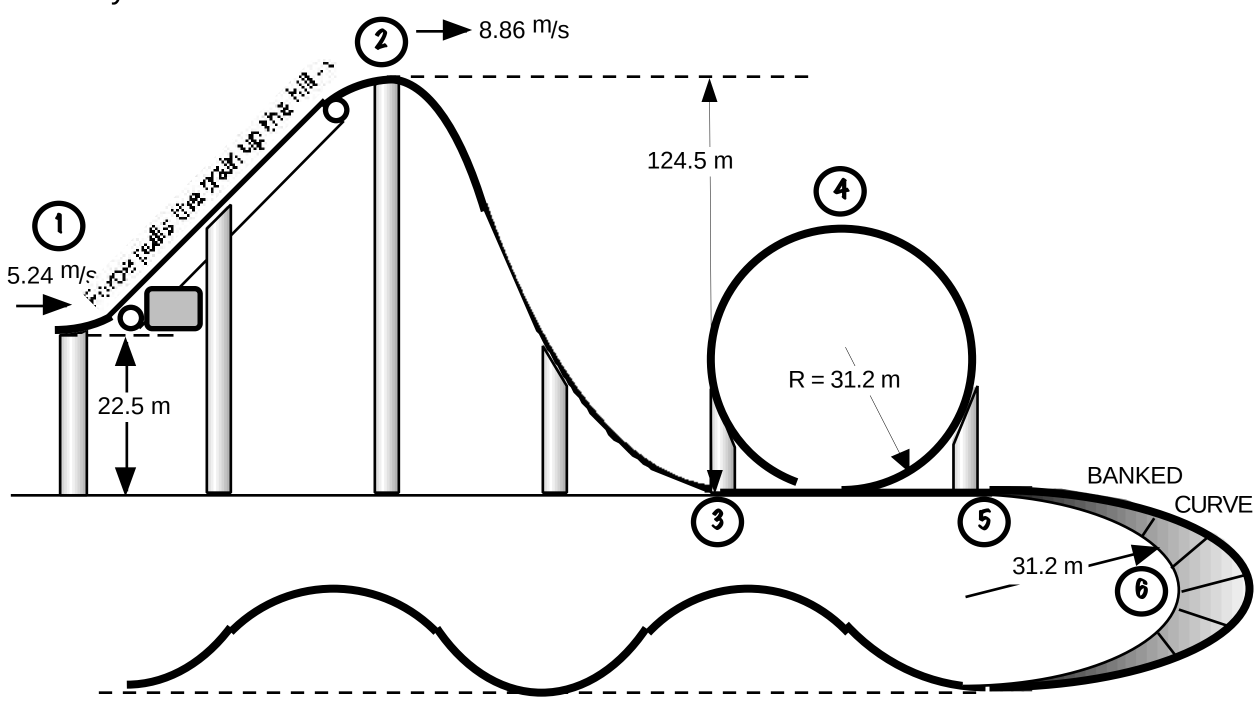
Because of the need for a larger radius as the rider enters the loop above, this coaster might be a good candidate for an irregular loop like the one below.



For this new loop, the designer will have to calculate the new velocity at the top of the loop using the new height. The g's felt will also need to be recalculated using the 2 new radii. This is left as an exercise for the reader.

STEP 6 The banked curve.

The banked curve is a horizontal curve on the ground in this diagram. Because it is at the lowest point it's velocity is equal to that of location #3, where v = 50.084 m/s. For a first try I'll make the radius 31.2 m.



The curve will be designed at the optimum angle where no friction or outside lateral forces are needed to keep the car on the track at speed. "At speed" means the velocity of the track design.

$$\tan(\theta) = \frac{v^2}{rg}$$

$$\tan(\theta) = \frac{50.084^2 \text{ m/s}}{31.2 \text{ m}(9.8) \text{ m/s}^2)}$$

$$\tan(\theta) = 0.164$$

$\theta = 9.3°$ .... That's almost a flat turn. It might be more exciting to try to decrease the radius so a greater banking angle will be needed.

The g's felt are calculated from

$$\text{g's felt} = \frac{1}{\cos(\theta)}$$

$$\text{g's felt} = \frac{1}{\cos(9.3°)}$$

g's felt = 1.013 g's .... This is not much more than normal gravity. These g's are the g's felt applied to your seat. Because this curve is rather flat, it would be wise to examine lateral, centripetal, acceleration in g's.

Lateral g's are the g's felt in the horizontal plain of the curve.

$v = 50.084$ m/s

$r = 31.2$ m ... of the curve. This just happens to be the same as the loop's radius.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{50.084^2 \text{ m/s}}{31.2 \text{ m}}$$

$$a_c = 80.398 \text{ m/s}^2$$

$$a_c = \frac{80.398 \text{ m/s}^2}{9.80 \text{ m/s}^2}$$

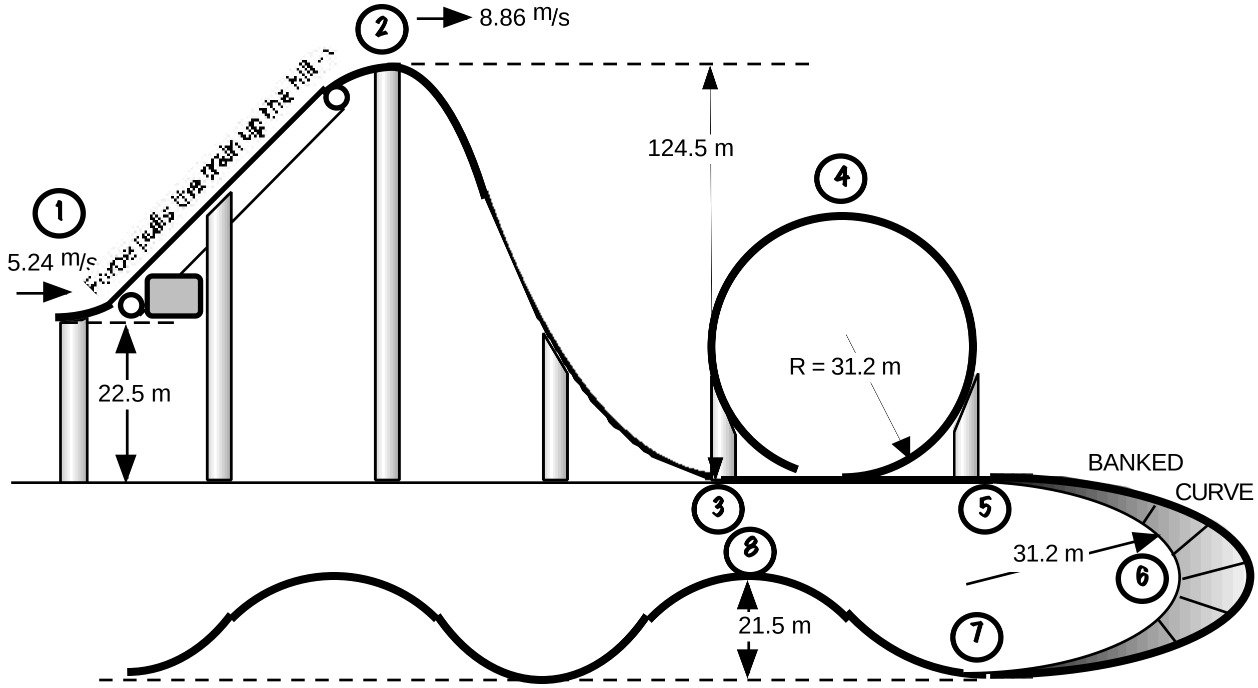
$a_c = 8.2$ g's ... Do not add or subtract a "g" because the circular motion is horizontal and not in the vertical plain.

... That is a lot of lateral g's. That's 8.2 times the rider's weight pressing him against the side of the coaster car. It is too extreme. Maybe a value around 1 to 2 g's would be better tolerated by the rider.

The banked curve needs to be redesigned.

STEP 7 The camel back

The camel back humps begin at the lowest part of the track and climb to a height of 21.5 m. The calculations to check the velocity at the top of the hump is similar to the one for the drop from location 2 to 3.



$$ET_{(LOCATION\ \#7)} = ET_{(LOCATION\ \#8)}$$

$$KE + PE = KE + PE$$

$$(1/2)mv^2 + 0 = (1/2)mv^2 + mgh$$

$$(1/2)v^2 + 0 = (1/2)v^2 + gh$$

$$(1/2)(50.084)^2 + 0 = (1/2)(v)^2 + (9.8)(21.5)$$

$$1254.203528 = (1/2)(v)^2 + 210.7$$

$$2087.007056 = (v)^2$$

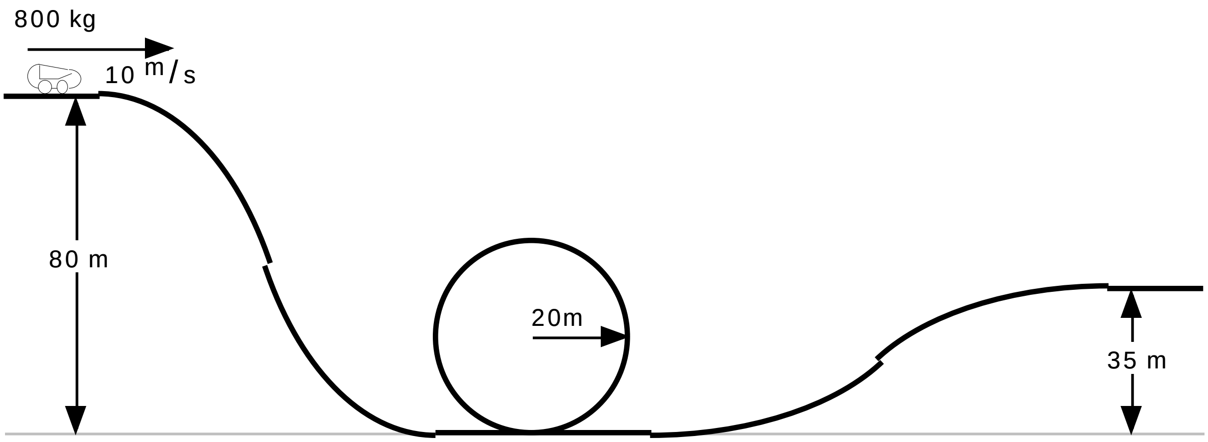
$$v = 45.6838$$

$\underline{v = 45.7}$ m/s ... At the bottom of the first camel back hill.

That's 102 mi/hr !!!

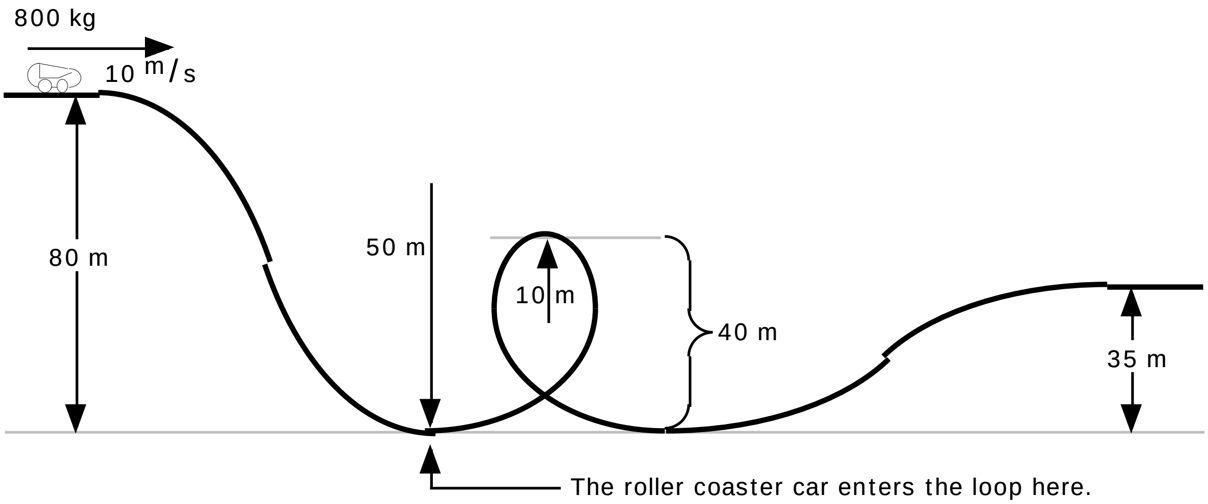
It might be wise to decrease the height of the first drop to decrease the velocity of the coaster as it enters the loop and the banked curve. Then, redesign the loop and banked curve to reduce g's experienced by the rider. As for the camel back humps at the end of the ride, they probably need to either be taller or stretched out horizontally. They are too narrow, horizontally, for 45.7 m/s.

STANDARD LOOP COASTER



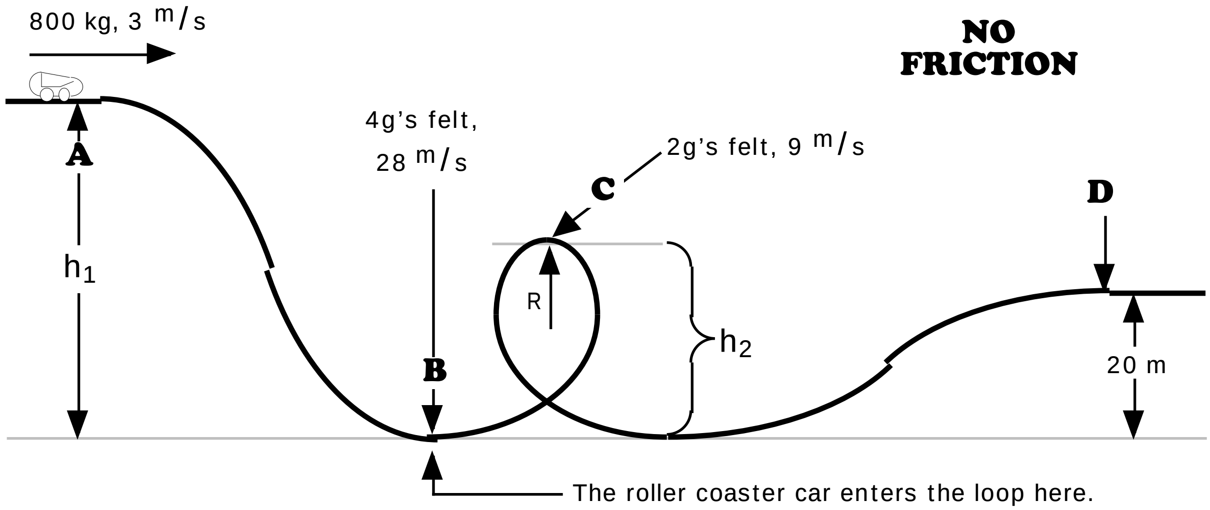
1 How fast is the roller coaster car traveling at the bottom of the hill?
2 How fast is the roller coaster traveling as it enters the loop?
3 What is the centripetal acceleration applied by the track at the bottom of the loop?
4 How many g's does the rider feel at the bottom of the loop?
5 How fast is the roller coaster car traveling at the top of the loop?
6 What is the centripetal acceleration applied by the track at the top of the loop?
7 How many g's does the rider feel at the top of the loop?
8 How fast is the roller coaster car traveling at the top of the 35 m hill?

IRREGULAR LOOP COASTER



1. How fast is the roller coaster car traveling at the bottom of the hill?
2. How fast is the roller coaster traveling as it enters the loop?
3. What is the centripetal acceleration applied by the track at the bottom of the loop?
4. How many g's does the rider feel at the bottom of the loop?
5. How fast is the roller coaster car traveling at the top of the loop?
6. What is the centripetal acceleration applied by the track at the top of the loop?
7. How many g's does the rider feel at the top of the loop?
8. How fast is the roller coaster car traveling at the top of the 35 m hill?

ROLLER COASTER DETECTIVE



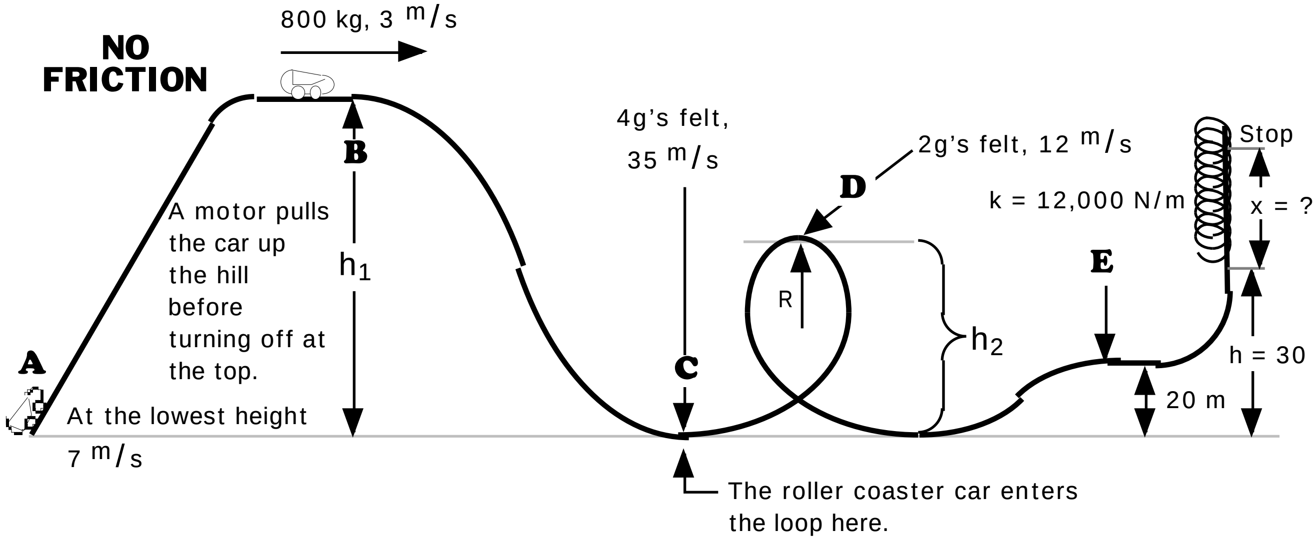
1 What is the height, between "**A**" and "**B**?"
2 What is the centripetal acceleration at "**B**?"
3 What is the radius of the track at "**B**?"
4 How high is location "**C**?"
5 What is the radius of the track at location "**C**?"
6 What is the velocity of the car at location "**D**?"

ANSWERS

1) 39.54 m **2**) 29.4 $^{m}/_{s^2}$ **3**) 26.67 m **4**) 35.87 m **5**) 2.76 m **6**) 19.80 $^{m}/_{s}$

ROLLER COASTER DETECTIVE

ADVANCED SHEET



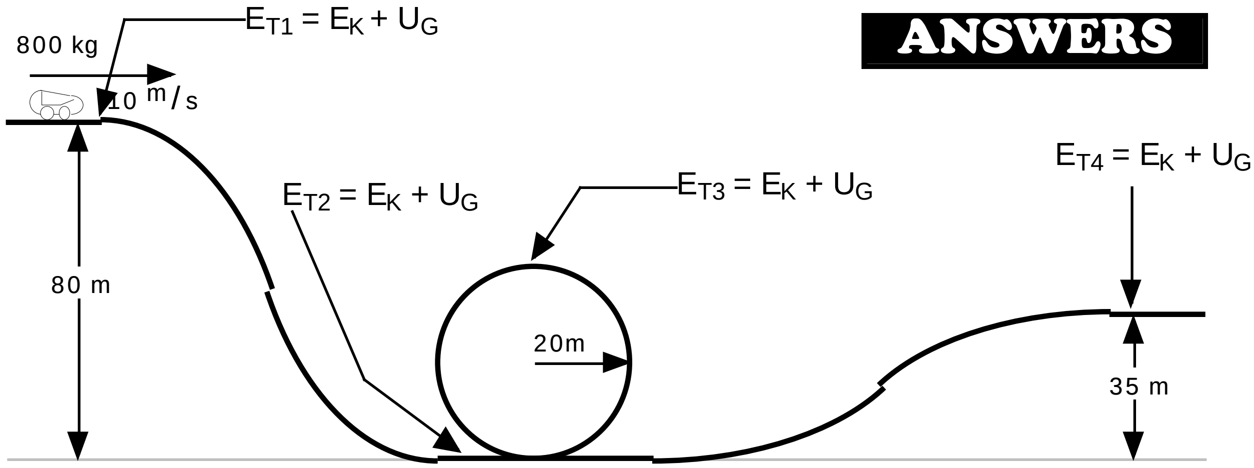
1 What is the height, between "**B**" and "**C**?"
2 What is the centripetal acceleration at "**C**?"
3 What is the radius of the track at "**C**?"
4 How high is location "**D**?"
5 What is the radius of the track at location "**D**?"
6 How fast is the car traveling at location "**E**?"
7 How far is the spring compressed from its hanging position, x?
8 If the car traveled up the first hill in 1.2 minutes, then how much power was used to pull it up the hill in horsepower?

ANSWERS

1) 62.04 m **2**) 29.4 m/s^2 **3**) 41.7 m **4**) 55.15 m **5**) 7.34 m **6**) 28.86 m/s **7**) 5.89 m **8**) 8.76 hp, 6533 w

STANDARD LOOP COASTER

ANSWERS



1. $E_{T1} = E_{T2}$

$E_K + U_G = E_K + U_G$

$1/2\ mv^2 + mgh = 1/2\ mv^2$

$1/2\ v^2 + gh = 1/2\ v^2$

$1/2\ 10^2 + 9.8(80) = 1/2\ v^2$

$50 + 784 = 1/2\ v^2$

$1668 = v^2$

$v = 40.84\ ^m/_s$

2. 40.84 $^m/_s$ because it's height does not change from #1.

3. $A_C = \frac{v^2}{R}$

$A_C = \frac{40.84^2}{20}$

$A_C = 83.4\ ^m/_{s^2}$

4. A_C in g's = $\frac{83.4\ ^m/_{s^2}}{9.8\ ^m/_{s^2}}$

A_C = 8.51 g's

Add 1 g at the bottom for what the rider feels.

8.51 g + 1 g = 9.51 g's

5. $E_{T1} = E_{T3}$

$E_K + U_G = E_K + U_G$

$1/2\ mv^2 + mgh = 1/2\ mv^2 + mgh$

$1/2\ v^2 + gh = 1/2\ v^2 + gh$

$1/2(10)^2 + 9.8(80) = 1/ + 9.8(40)$

$50 + 784 = 1/2\ v_2 + 392$

$442 = 1/2\ v^2$

$v = 29.73\ ^m/_s$

6. $A_C = \frac{v^2}{R}$

$A_C = \frac{29.73^2}{20}$

$A_C = 44.2\ ^m/_{s^2}$

A_C in g's = $\frac{44.2\ ^m/_{s^2}}{9.8\ ^m/_{s^2}}$

7. A_C in g's = 4.5 g's

Subtract 1 g at the top for what the rider feels.

4.5 g - 1 g = 5.5 g's

8. $E_{T1} = E_{T4}$

$E_K + U_G = E_K + U_G$

$1/2\ mv^2 + mgh = 1/2\ mv^2 + mgh$

$1/2\ v^2 + gh = 1/2\ v^2 + gh$

$1/2\ (10)^2 + 9.8(80) = 1/2\ v^2 + 9.8(35)$

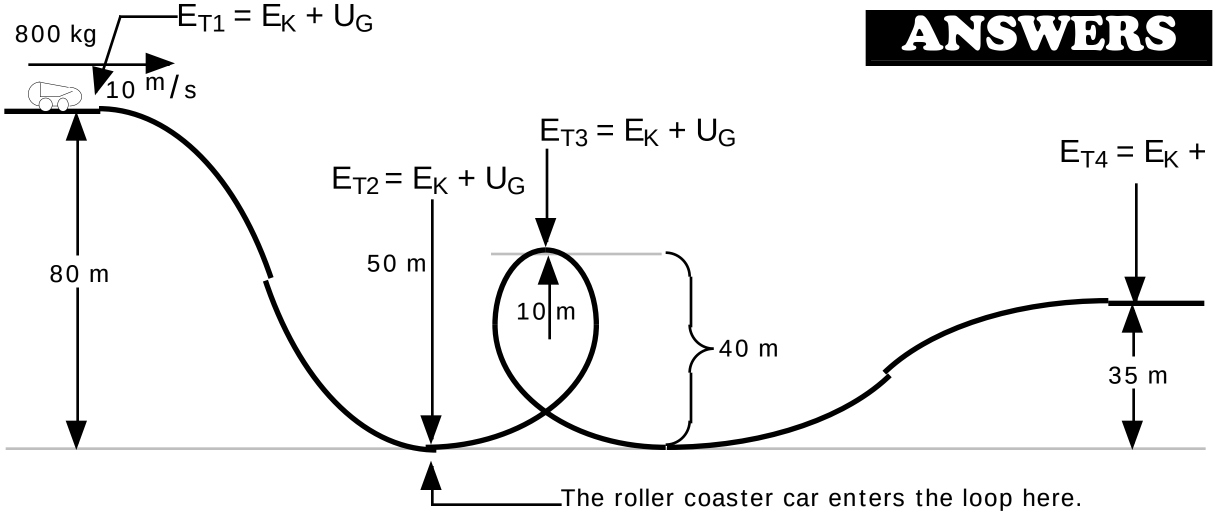
$50 + 784 = 1/2\ v^2 + 343$

$491 = 1/2\ v^2$

$v = 31.34\ ^m/_s$

IRREGULAR LOOP COASTER

ANSWERS



1. $E_{T1} = E_{T2}$

$E_K + U_G = E_K + U_G$

$\frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2$

$\frac{1}{2} v^2 + gh = \frac{1}{2} v^2$

$\frac{1}{2} 10^2 + 9.8(80) = \frac{1}{2} v^2$

$50 + 784 = \frac{1}{2} v^2$

$1668 = v^2$

$v = 40.84$ m/s

2. 40.84 m/s because it's height does not change from #1.

3. $A_C = \frac{v^2}{R}$

$A_C = \frac{40.84^2}{50}$

$A_C = 33.36$ m/s²

4. A_C in g's $= \frac{33.36 \text{ m/s}^2}{9.8 \text{ m/s}^2}$

$A_C = 3.40$ g's

Add 1 g at the bottom for what the rider feels.

3.40 g + 1 g = 4.40 g's

5. $E_{T1} = E_{T3}$

$E_K + U_G = E_K + U_G$

$\frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2 + mgh$

$\frac{1}{2} v^2 + gh = \frac{1}{2} v^2 + gh$

$\frac{1}{2}(10)^2 + 9.8(80) = \frac{1}{2}v^2 + 9.8(40)$

$50 + 784 = \frac{1}{2} v_2 + 392$

$442 = \frac{1}{2} v^2$

$v = 29.73$ m/s

6. $A_C = \frac{v^2}{R}$

$A_C = \frac{29.73^2}{10}$

$A_C = 88.39$ m/s²

A_C in g's $= \frac{88.39 \text{ m/s}^2}{9.8 \text{ m/s}^2}$

7. A_C in g's = 9.02 g's

Subtract 1 g at the top for what the rider feels.

9.02 g - 1 g = 8.02 g's felt

8. $E_{T1} = E_{T4}$

$E_K + U_G = E_K + U_G$

$\frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2 + mgh$

$\frac{1}{2} v^2 + gh = \frac{1}{2} v^2 + gh$

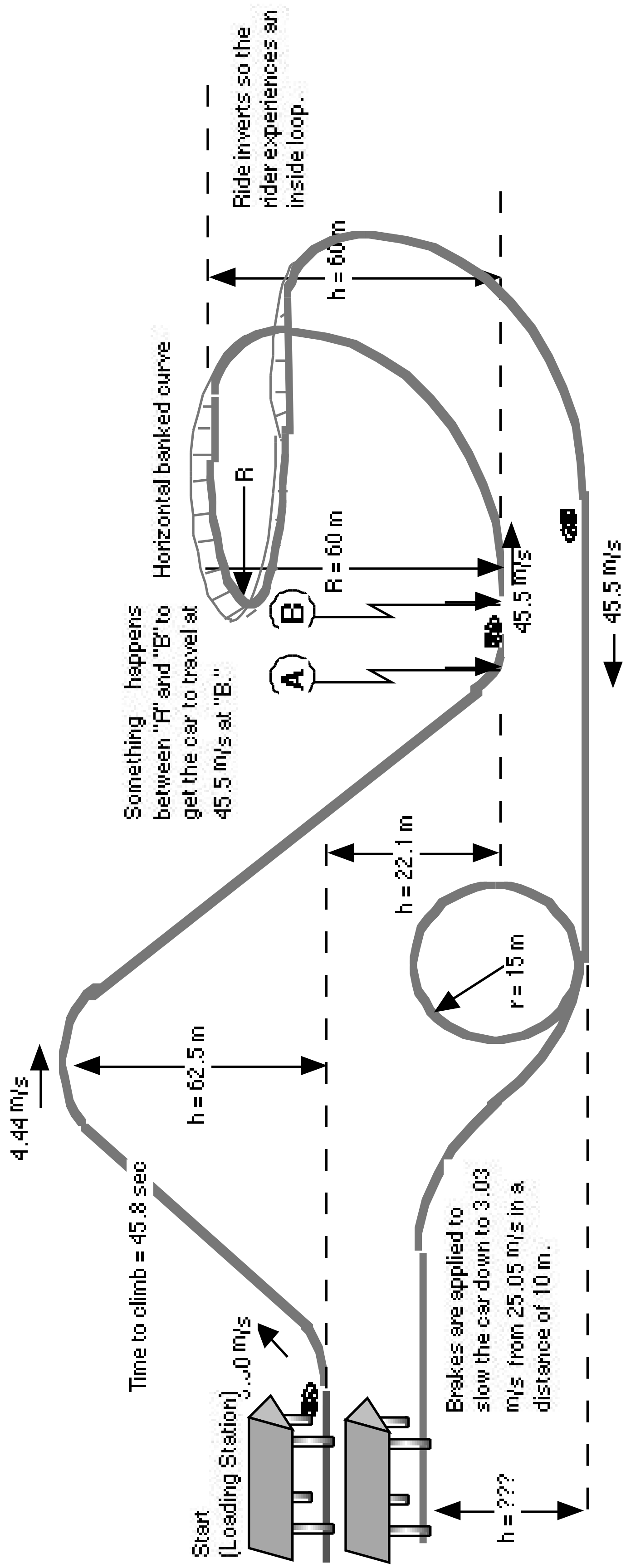
$\frac{1}{2} (10)^2 + 9.8(80) = \frac{1}{2} v^2 + 9.8(35)$

$50 + 784 = \frac{1}{2} v^2 + 343$

$491 = \frac{1}{2} v^2$

$v = 31.34$ m/s

The information on this diagram is to be used for the accompanying questions.



Using the information shown on the diagram on the other page, determine the correct answers to the following questions.

1. How much power was used to pull the coaster car up the first hill? (in watts and horsepower.)
2. How fast is the car traveling at location “A” if it coasted down the back side of the first hill?
3. How many g’s does the rider feel as he enters the bottom of the next element if it’s radius is 60.0 m, at location “B?”
4. How fast is the car traveling when it reaches the banked curve?
5. If the banked curve is at a 52.3° angle with the horizontal, then what is the radius of the banked curve?
6. How many g’s does the rider feel in the banked curve?
7. How many g’s does a rider feel as he enters the bottom of the loop?
8. How fast is the rider traveling at the top of the loop?
9. How many g’s does the rider feel at the top of the loop?
10. As the car enters the station at the end of the ride, its brakes are applied. How much power do the brakes exert on the car? (in watts and horsepower.)

ANSWERS

1 Use Energy relationships and power to solve

$v_o = 5.50$ m/s
$v_f = 4.44$ m/s
h = 62.5 m
t = 45.8 s
m =6500 kg

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{\Delta\text{Energy}}{\text{time}}$$

$\Delta ET = [KE_f + PE_f] - [KE_o + PE_o]$
$\Delta ET = [(1/2)m(v_f)^2 + mg(h_f)] - [(1/2)m(v_o)^2 + 0]$
$\Delta ET = [(1/2)(6500)(4.44)^2 + (6500)(9.80)(62.5)] - [(1/2)(6500)(5.50)^2]$
$\Delta ET = 64069.2 + 3981250 - 98312.5$
$\Delta ET = 3947006.7$ J

$$\text{Power} = \frac{\Delta\text{Energy}}{\text{time}} = \frac{3947006.7\text{ J}}{45.8\text{ s}}$$

Power = 86,179.186 watts ...746 w = 1 hp
Power = 115.52 hp

2 Use Energy relationships to solve

$v_o = 4.44$ m/s ...at the hill top
$v_f = ????$ ...at the hill bottom
h = 62.5 + 22.1 = 84.6 m
ET @ the hill top = ET @ the hill bottom
KE + PE = KE + PE
$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$
$(1/2)v^2 + gh = (1/2)v^2 + 0$
$(1/2)(4.44)^2 + (9.80)(84.6) = (1/2)v^2$
$838.94 = (1/2)v^2$
$v = 40.96$ m/s

3 Use circular motion relationships to solve

v = 45.5 m/s, r = 60.0 m

$$a_c = \frac{V^2}{r} = \frac{45.5^2}{60.0} = 34.50417\ \text{m/s}^2$$

$$a_c = \frac{34.50417\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 3.52\text{ g's ...convert } a_c \text{ to g's}$$

g's felt at the bottom = a_c[in g's] + 1 g
g's felt at the bottom = 3.52 + 1 g
g's felt at the bottom = 4.52 g's

4 Use Energy relationships to solve

ET @ "B" = ET @ "A"

KE + PE = KE + PE

$(\frac{1}{2})mv^2 + 0 = (\frac{1}{2})mv^2 + mgh$

$(\frac{1}{2})v^2 = (\frac{1}{2})v^2 + gh$

$(\frac{1}{2})(45.5)^2 = (\frac{1}{2})v^2 + (9.80)(60.0)$

$447.125 = (\frac{1}{2})v^2$

$\underline{v = 29.904}$ m/s

5 Use banked curves

v = 29.90401311 m/s ...from #4

θ = 52.3°

$$\tan(\theta) = \frac{v^2}{(rg)}$$

$$\tan(52.3°) = \frac{(29.90401311)^2}{(R)(9.80)}$$

R = 70.52601458 m

R = 70.5 m

6 Use banked curves

$$\text{g's felt} = \frac{1}{\sin(\theta)}$$

$$\text{g's felt} = \frac{1}{\sin(52.3°)}$$

g's felt = 1.26

7 Use circular motion relationships to solve

v = 45.5 m/s ...because it is at the same height as location "B."

r = 15 m

$$a_c = \frac{V^2}{r} = \frac{45.5^2}{15} = 138.01667 \text{ m/s}^2$$

$$a_c = \frac{138.01667 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 314.083 \text{ g's} \text{ ...convert } a_c \text{ to g's}$$

g's felt at the bottom = a_c[in g's] + 1 g

g's felt at the bottom = 14.083 + 1 g

<u>g's felt at the bottom = 15.0 g's</u>

8 Use Energy relationships to solve

ET @ bottom = ET @ top

KE + PE = KE + PE

$(1/2)mv^2 + 0 = (1/2)mv^2 + mgh$

$(1/2)v^2 = (1/2)v^2 + gh$

$(1/2)(45.5)^2 = (1/2)v^2 + (9.80)(30.0)$

$741.125 = (1/2)v^2$

$\underline{v = 38.5}$ m/s

9 Use circular motion relationships to solve

v = 38.5 m/s ...from #8

r = 15 m

$$a_c = \frac{V^2}{r} = \frac{38.5^2}{15} = 98.8167\ \text{m/s}^2$$

$$a_c = \frac{98.8167\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 10.083 \text{ g's}$$ ...convert a_c to g's

g's felt at the top = a_c[in g's] - 1 g

g's felt at the top = 10.083 - 1 g

<u>g's felt at the top = 9.08 g's</u>

10 Use energy relationships and power to solve

$v_o = 25.05$ m/s

$v_f = 3.03$ m/s

d = 10 m

t = 45.8 s

m =6500 kg

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{\Delta\text{Energy}}{\text{time}}$$

$\Delta ET = [KE_f + PE_f] - [KE_o + PE_o]$

$\Delta ET = [(1/2)m(v_f)^2 + 0] - [(1/2)m(v_o)^2 + 0]$

$\Delta ET = (1/2)(6500)(25.05)^2 - (1/2)(6500)(5.50)^2$

$\Delta ET = 2010103.875J$

$$\frac{\text{distance}}{\text{time}} = \frac{\text{(final velocity)} + \text{(initial velocity)}}{2}$$

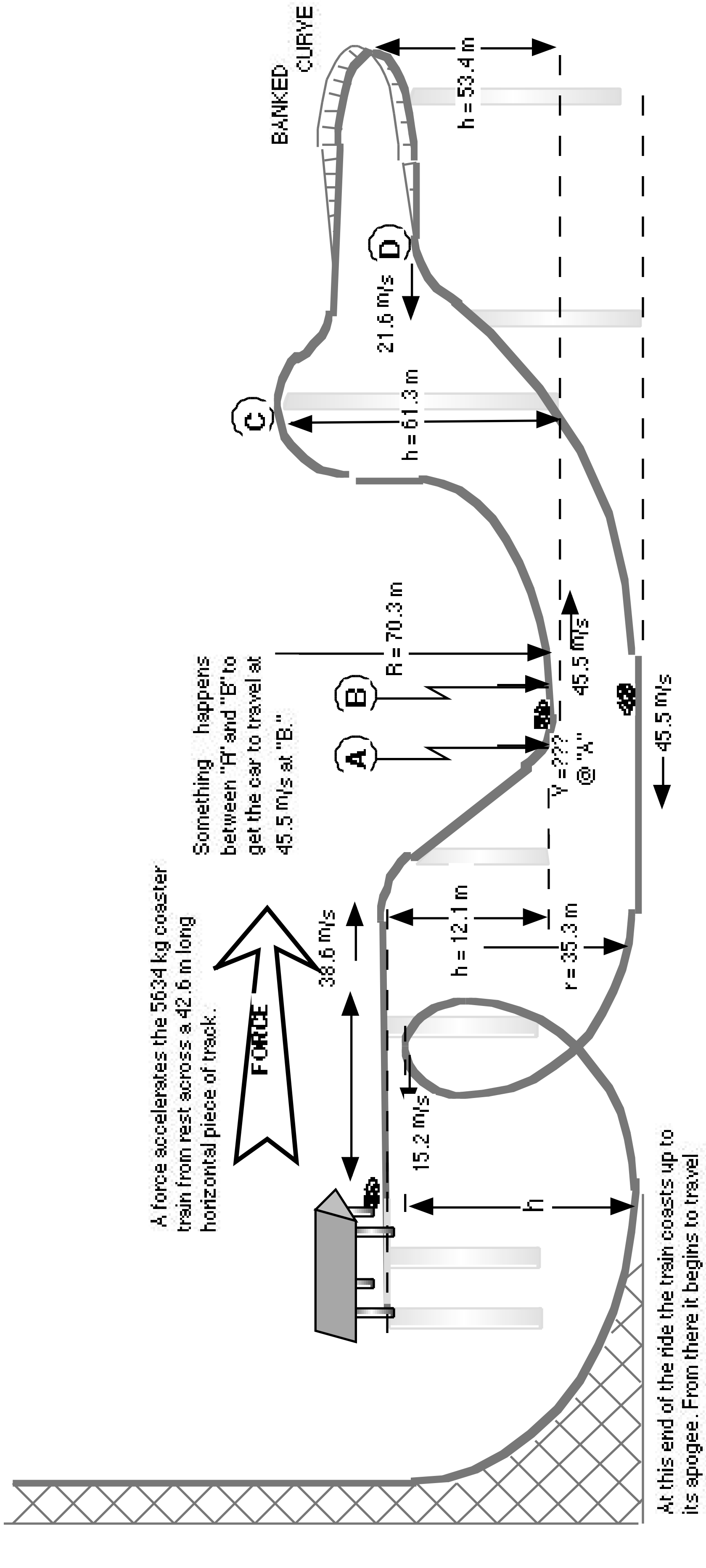
$$\frac{10}{t} = \frac{25.03 + 3.03}{2}$$

t = 0.712250712 s

$$\text{Power} = \frac{\Delta\text{Energy}}{\text{time}} = \frac{2010103.875}{0.712250712}$$

Power = 2822185.841watts ...746 w = 1 hp

<u>Power = 3783.09 hp</u>


BANKED CURVE
h = 53.4 m
D
21.6 m/s
C
h = 61.3 m
R = 70.3 m
B
A
45.5 m/s
V = ??? @ "X"
45.5 m/s
Something happens between "A" and "B" to get the car to travel at 45.5 m/s at "B."
A force accelerates the 5634 kg coaster train from rest across a 42.6 m long horizontal piece of track.
FORCE
38.6 m/s
h = 12.1 m
r = 35.3 m
15.2 m/s
h
At this end of the ride the train coasts up to its apogee. From there it begins to travel along the track backwards.

Using the information shown on the diagram on the other page, determine the correct answers to the following questions.

1. How much force is used to accelerate the train across the first 42.6 m?
2. How many g's does the rider feel as he is pushed by the initial accelerating force?
3. How fast is the car traveling at location "A" if it coasted down the back side of the first hill?
4. At location "B" the incline is 90° of a circle. How many g's does the rider feel as he enters the loop at location "B?"
5. How fast is the car traveling when it reaches location "C"?
6. In the banked turn the rider is traveling 21.6 m/s. What is the optimum angle of the banked curve?
7. How many g's does the rider feel as he enters the loop?
8. What is the height of the loop?
9. If the radius at the top of the loop is 7.45 m, then how many g's does the rider feel at this location?
10. How high does the coaster train coast at the end of the track?

ANSWERS

1 Use energy relationships to solve

$ET_{(BEGINNING)} + WORK = ET_{(BEGINNING)}$

$0 + Fd = (1/2)\ mv^2$

$F(42.6) = (1/2)\ 5634)(38.6)^2$

$F(42.6) = 4197217.32$

$F = 98{,}526.22817$ N

$\underline{F = 98{,}500 \text{ N}}$

2 Use kinematics to solve

$v_f^2 = v_o^2 + 2ax$

$38.6^2 = 0 + 2a(42.6)$

$a_c = 17.48779\ ^m/_{s^2}$

$a_c = \dfrac{17.48779\ ^m/_{s^2}}{9.80\ ^m/_{s^2}}$ convert a_c to g's

3 Use energy relationships to solve

$ET_{(AFTER\ FORCE)} = ET_{\text{(at "A")}}$

$KE + PE = KE + PE$

$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$

$(1/2)v^2 + gh = (1/2)\ v^2 + 0$

$(1/2)(36.8)^2 + 9.8(12.1) = (1/2)v^2$

$863.56 = (1/2)v^2$

$1727.12 = v^2$

$v = 41.55863\ ^m/_s$

$\underline{v = 41.6}\ ^m/_s$

4 v = 45.5 m/s, r = 70.3 m

$a_c = \dfrac{V^2}{r} = \dfrac{45.5^2}{70.3} = 29.44879\ ^m/_{s^2}$

$a_c = \dfrac{29.44879\ ^m/_{s^2}}{9.80\ ^m/_{s^2}} = 3.0049787$ g's ...convert a_c to g's

g's felt at the bottom = a_c[in g's] + 1 g

g's felt at the bottom = 3.0049787 + 1 g

<u>g's felt at the bottom = 4.00 g's</u>

5 Use energy relationships and compare locations "B" with "C"

$$ET_B = ET_C$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + 0 = (1/2)mv^2 + mgh$$
$$(1/2)v^2 + 0 = (1/2)v^2 + gh$$
$$(1/2)(45.5)^2 + 0 = (1/2)v^2 + 9.8(61.3)$$
$$1035.125 = (1/2)v^2 + 600.74$$
$$868.77 = v^2$$
$$v = 29.4749\ m/s$$
$$\underline{v = 29.5}\ m/s$$

6 Use banked curve equations

$v = 21.6\ m/s$, $r = 42.3$ m

$$\tan(\theta) = \frac{v^2}{(rg)}$$
$$\tan(\theta) = \frac{21.6^2}{(42.6)(9.80)}$$
$$=\theta\ 48.117768°$$
$$\underline{\theta = 48.1°}$$

7 Use circular motion equations

$v = 45.5\ m/s$, $r = 35.3$ m

$$a_c = \frac{V^2}{r} = \frac{45.5^2}{35.3} = 58.6473\ m/s^2$$

$$a_c = \frac{58.6473\ m/s^2}{9.80\ m/s^2} = 5.984419 \text{ g's ...convert } a_c \text{ to g's}$$

g's felt at the bottom = a_c[in g's] + 1 g
g's felt at the bottom = 5.984419 + 1 g
<u>g's felt at the bottom = 6.98g's</u>

8 Use energy relationships

$v_{(BOTTOM)} = 45.5\ m/s$, $V_{(TOP)} = 15.2\ m/s$

$$ET_{(TOP)} = ET_{(BOTTOM)}$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$$
$$(1/2)v^2 + gh = (1/2)v^2$$
$$(1/2)(15.2)^2 + (9.80)h = (1/2)(45.5)^2$$
$$115.52 + 9.8(h) = 1035.125$$
$$h = 93.8372\ m$$
$$\underline{h = 93.8\ m}$$

9 Use circular motion relationships

$v = 15.2\ ^{m}/_{s}$, $r = 7.45$ m

$$a_c = \frac{V^2}{r} = \frac{15.2^2}{7.45} = 31.01208\ ^{m}/_{s^2}$$

$$a_c = \frac{31.01208\ ^{m}/_{s^2}}{9.80\ ^{m}/_{s^2}} = 3.164498 \text{ g's} \quad \text{...convert } a_c \text{ to g's}$$

g's felt at the top of a loop = a_c[in g's] + 1 g

g's felt at the top of a loop = 3.164498 + 1 g

<u>g's felt at the top of a loop= 2.16 g's</u>

10 Use energy relationships to solve.

Velocity at the highest point will be zero, (apogee).

$$ET_{(BOTTOM)} = ET_{(TOP)}$$

$$KE + PE = KE + PE$$

$$(^1/_2)mv^2 + 0 = 0 + mph$$

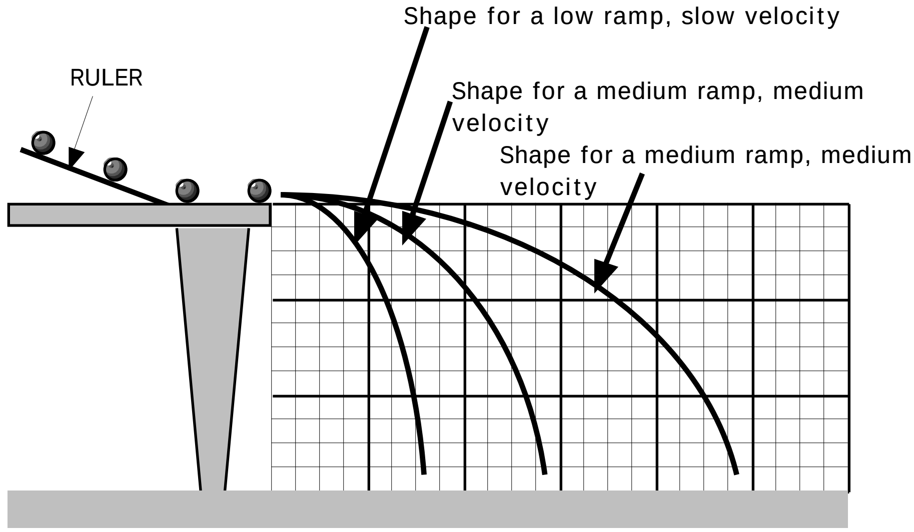
$$(^1/_2)v^2 = gh$$

$$(^1/_2)(45.5)^2 + \ = (9.80)h$$

$$h = 105.625 \text{ m}$$

<u>h = 106 m</u>

HILL AND DIPS: ACTIVITY 1

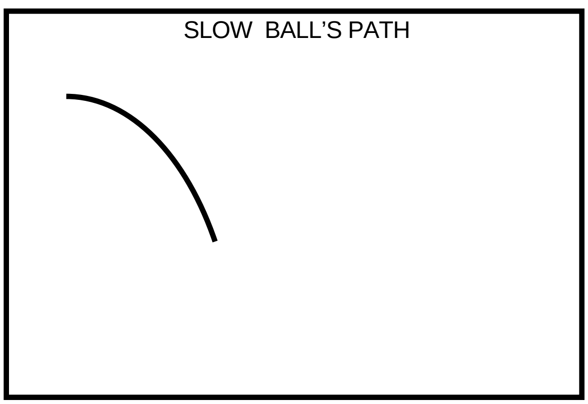


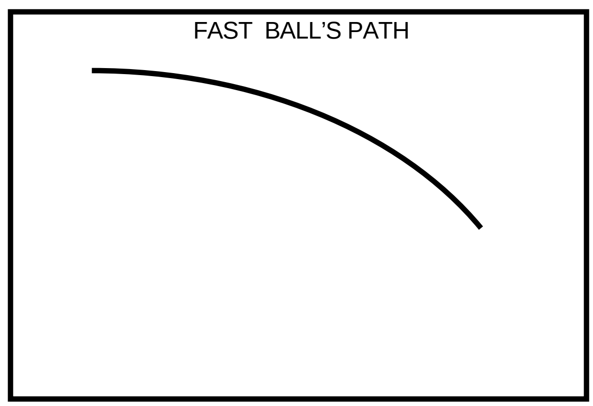
1. The steeper ramp yields the greatest velocity when leaving the table.

2. The greater the speed over the hill the flatter, more spread out, the hill must become. See page 22 in the text for the other parts of the solution.

HILL DESIGN: ACTIVITY 1

Draw the results below.





HILL DESIGN: ACTIVITY 2

1-3 For the hill where the car becomes airborne, the hill width will be shorter than the hill where the car does not become airborne.

4 The car will become airborne over the hill because it will be traveling too fast. And the hill needs to be the same shape as if there were no track, a parabola.

5 The car will become airborne over hill B. The two hills are the same height. The steeper the drop off on the right side of the hill the slower the car needs to go. This is because hill B is the slow ball's path (shown in Activity 1.)

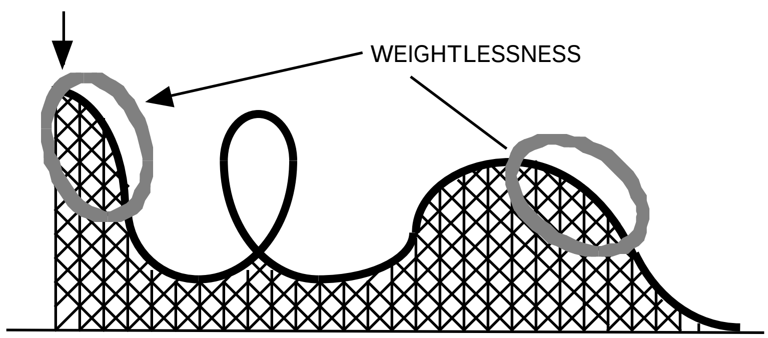
LOOP DESIGN: ACTIVITY 1

The starting height for the klothoid loop will be lower than the starting height for the circular loop. The advantage of the klothoid loop's design is that the roller coaster car does not have to be going as fast to stay on the track when the car is upside down.

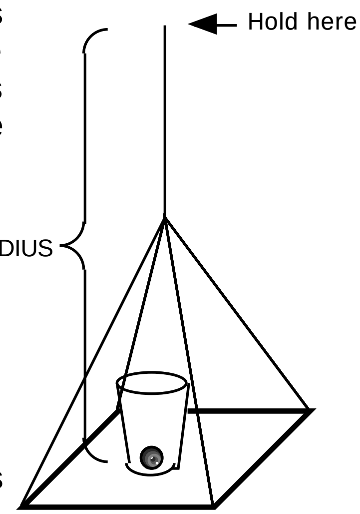
FREE FALL (Weightlessness): ACTIVITY 1

Why doesn't the water come out of the holes when it is dropped?

The water comes out of the holes when the cup is standing still and the water is allowed to accelerate down relative to the cup. When the cup is dropped, it is accelerating down at the same rate as the cup. The cup has no reaction force holding it up and the water stays in.



STAYING SEATED



Block this out when copying:
To calculate the centripetal acceleration you will need to know the radius the cup spins in and the time to go around once. The radius can be calculated while holding the cup and board still. Someone should swing the board around with as consistent a velocity as possible. Time how long it takes to go around 10 times. To find the time to go around once,

$$\text{Time to go around once} = \frac{\text{Time to go around 10 times}}{10}$$

to find the centripetal acceleration use the equation

$$a = \frac{4\pi^2 R}{T^2}$$

where "a" is the centripetal acceleration, "R" is the radius and "T" is the time to go around once.
To convert the acceleration to g's divide the acceleration by 9.8. (This is assuming your original measurements are in meters and seconds.)

EXAMPLE: A board connected to a rope whose length from the student's hand to the board is 0.50m. The board takes 10.77 seconds to go around 10 times.

SOLUTION: The time to go around once is 1.077 seconds. The centripetal acceleration is 17 m/s^2. This is 1.73 g's.

PRACTICING YOUR "ESTIMATIONS"

MEASURING THE VELOCITY OF A MOVING OBJECT