Roller Coaster Physics

An Educational Guide To Roller Coaster Design and Analysis for Teachers and Students

This booklet contains many supplemental science materials to be used in conjunction with a visit to an amusement park for middle and high school students.

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INTRODUCTION

This booklet will discuss some of the principles involved in the design of a roller coaster. It is intended for the middle or high school teacher. Physics students may find the information helpful as well. Many of the concepts can be applied to topics other than roller coasters. Some sections will use the "Roller Coaster Simulator," RCS. (See page 78 for instructions on its construction.) The included activities are hands on cookbook type. Each section includes background topics that should have been taught previously.

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This book assumes some rudimentary knowledge of physical science. It is a simplified view of what design considerations and science a mechanical/civil design engineer must know when designing a roller coaster.

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GENERAL

A roller coaster is a balance between safety and sensation. Naturally, the ride should be as safe as possible. After all, if the people are injured riding the coaster then there would be fewer repeat riders. Fewer repeat riders means a short life span for the coaster. On the other hand, passengers ride a coaster for the "death defying" thrill. The key to a successful coaster is to give the rider the sensation of speed and acceleration. It all comes down to speed control.

To achieve this, the hills, curves, dips, straight always, braking systems and loops are not randomly designed. They follow some simple rules of physics.

In order to understand what is going on, students must understand the difference between velocity and acceleration.

VELOCITY

Velocity describes how quickly an object changes it position. The higher the velocity the quicker an object travels between 2 locations. Phrases like, "...how fast..., how quickly," are used to describe velocity. Often the word speed is substituted for the word velocity in common usage. However, technically the two are different. Velocity is actually speed with direction. For example, "60 mph, west," is a velocity. "West" is the direction and "60 mph" is the speed. The units of velocity are in the form of

$$\text{Velocity} = \frac{\text{Units of Distance}}{\text{Units of Time}}$$

$$\text{Example} = \frac{\text{meter}}{\text{second}} \quad \frac{\text{mile}}{\text{hour}} \quad \frac{\text{furlong}}{\text{week}}$$

ACCELERATION

Acceleration describes how quickly an object changes its velocity. Phrases like, "...slow down..., ...speed up..., ...change speed... and change velocity..." are used to describe accelerations. If a student wants an easy way to determine if he is visualizing acceleration or a constant velocity *along a straight line* he only needs to ask himself one question, "Is the object slowing down or speeding up?" If the answer is "Yes," then it is accelerating. If the answer is "No" then it is moving with a constant velocity. The units of acceleration are in the form of

$$\text{Acceleration} = \frac{\text{Units of Distance}}{(\text{Units of Time})^2} = \frac{\text{Units of Velocity}}{\text{Units of Time}}$$

$$\text{Example} = \frac{\text{meter}}{\text{second}^2} \quad \frac{\text{meter}}{\text{second} \cdot \text{hour}^2} \quad \frac{\text{mile}}{\text{hour}^2} \quad \frac{\text{furlong}}{\text{week}^2}$$

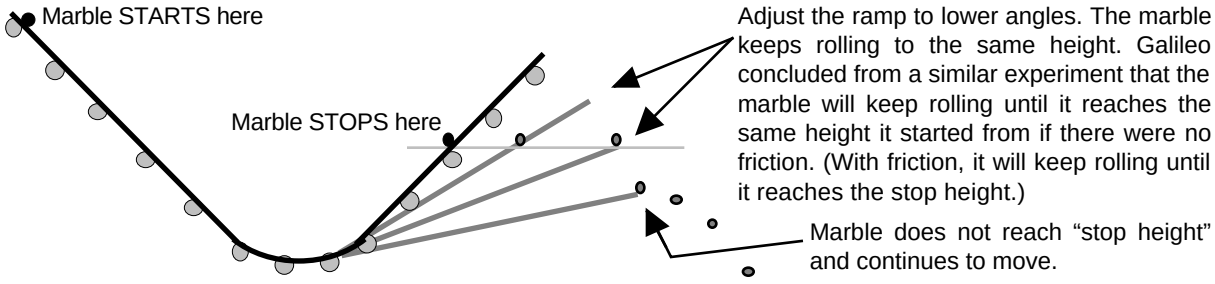
$$\left(\frac{\frac{\text{mile}}{\text{hour}}}{\text{sec}}\right) \quad \left(\frac{\frac{\text{meter}}{\text{sec}}}{\text{sec}}\right) \quad \left(\frac{\frac{\text{fathom}}{\text{min}}}{\text{sec}}\right)$$

The Simple Roller Coaster

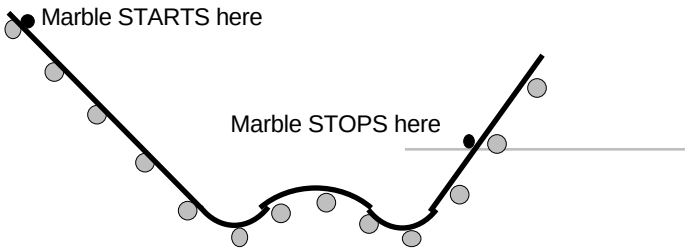
by Tony Wayne

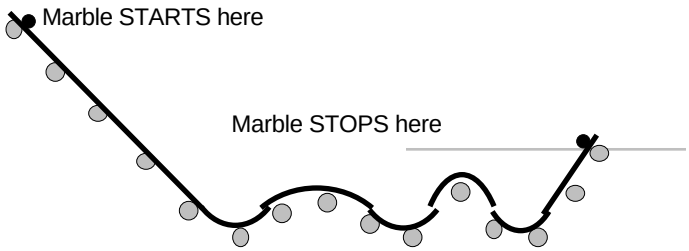
THE SIMPLE ROLLER COASTER

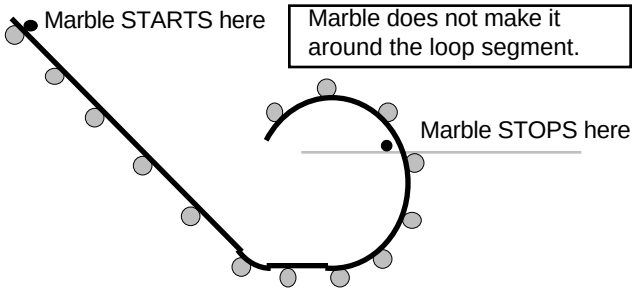
The simple roller coaster started with Galileo Galilei. Below is a description of how to demonstrate Galileo's experiment using the "Roller Coaster Simulator." [This experiment could also be duplicated using a HotWheels™ track and a marble."

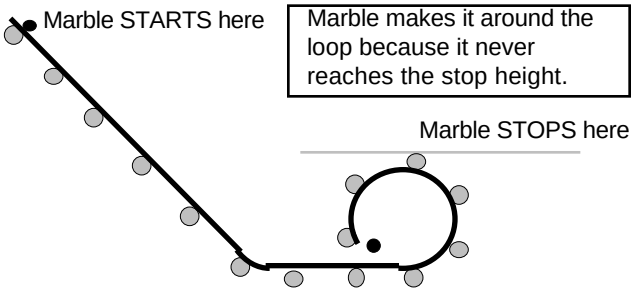


This has far-reaching implications. (1) The marble could take any path until it reaches the same height it starts from, assuming no friction. In the previous activity, the marble did not roll to the same height it started from because of friction. But it consistently rolls to the same height. To reflect these implications, the track could be reshaped as shown below.



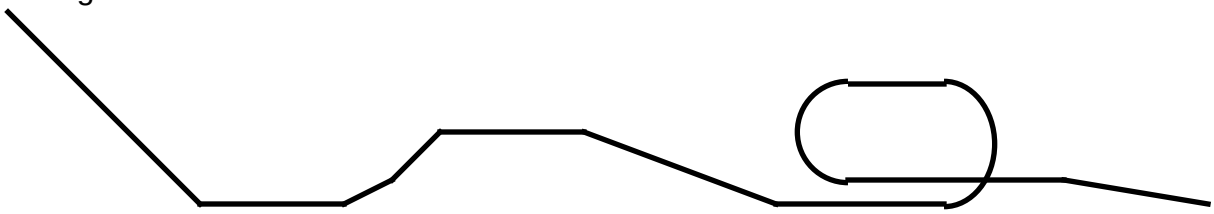






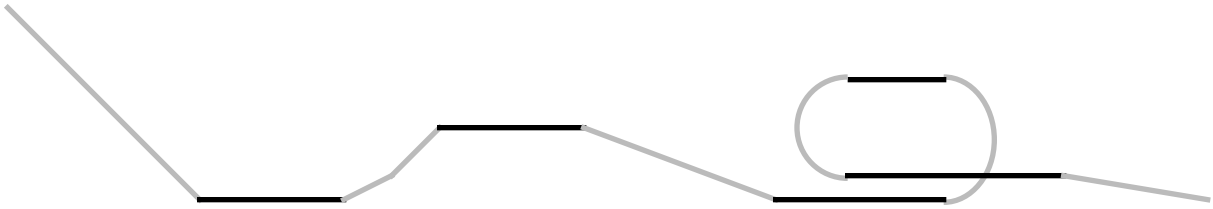
(2) The ball begins to roll down due to the force of gravity. It stops when all the energy gravity gave the ball is used up. The marble accelerates only while a force acts on it *in its direction of motion*.

Here is a good exercise to draw on a chalk board.



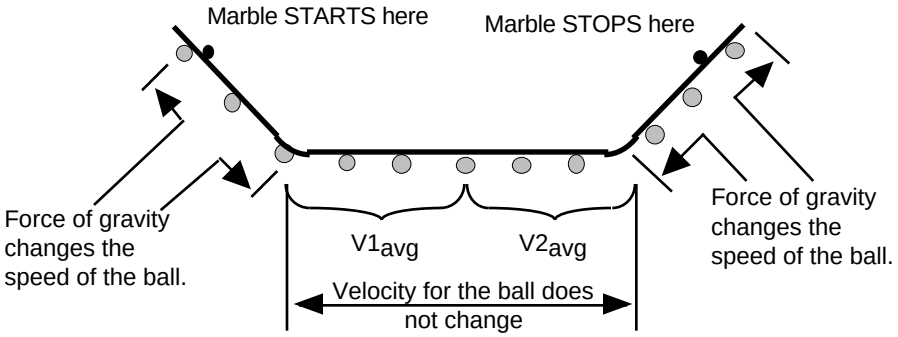
Across which sections of the track would a roller coaster car travel at a constant velocity or accelerate?

ANSWER



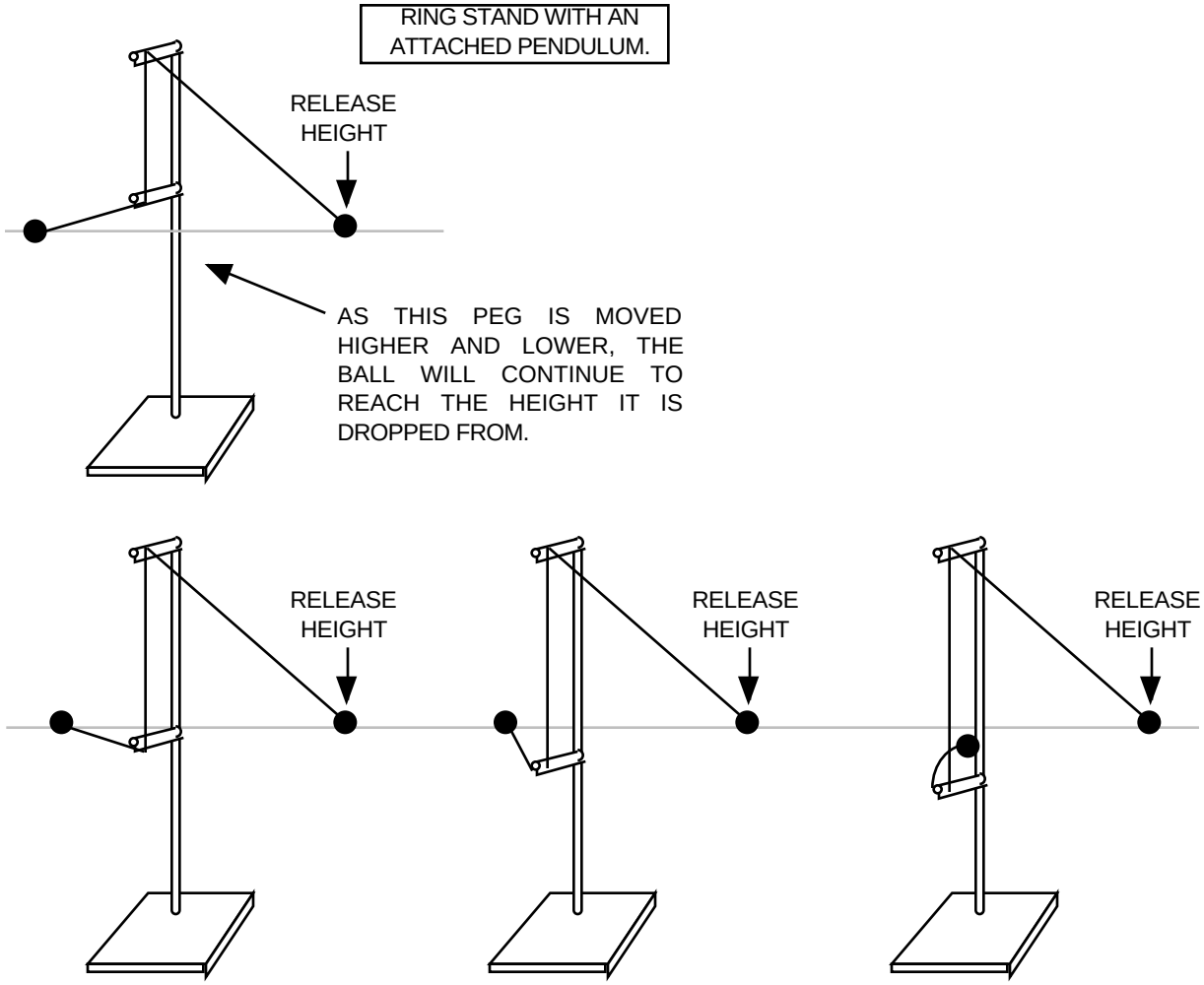
- ☐ The gray sections of track are where the coaster car accelerates. (Speeds up or slows down.)
- ■ The black sections of track are where the coaster car travels at a constant velocity.

The acceleration can be demonstrated experimentally using the roller coaster simulator or HotWheels™ track. If a long enough section is made horizontal, it can be shown that the average velocities calculated at the beginning and at the end of the horizontal section are equal. Form the track in the shape shown below. Roll a marble or steel bearing down the track. It will accelerate along the drop and move at a constant velocity along the horizontal section and slow down as it climbs up the opposite side. When the marble slows down and speeds up on the hills it is visually obvious. What is not so visually obvious is what happens along the horizontal section of the track. The ball's constant velocity can be shown mathematically. Divide the horizontal section of the track into 2 sections. Calculate the average velocity of the ball along these two sections. If done accurately, the velocities will be nearly equal. To obtain more accurate results, use fairly long sections of horizontal track. The longer the sections of track, the greater the time measurement. Longer time measurements mean lower percent errors.



Along the horizontal section of the track, ignoring the minimal effects of friction, there are no forces acting on the ball horizontally. Therefore the ball moves at a constant velocity while no force acts on it. This is Galileo's law of inertia!!!

Here is another example of an illustration of Galileo's experiment.



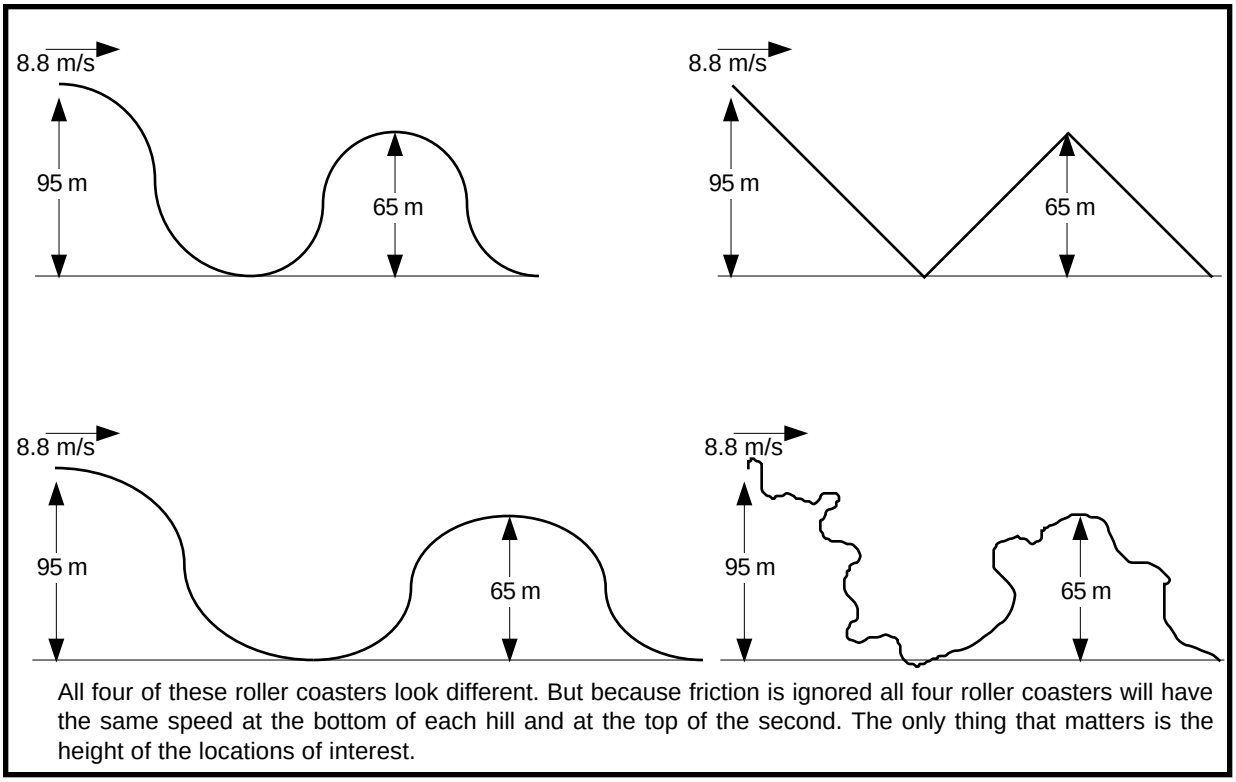
The ball continues until it reaches the starting height.

The Most Often Used Calculations

A roller coaster is called a coaster because once it starts it coasts through the entire track. No outside forces are required for most coasters. (A few have double or triple lift hills and braking sections.) Roller coasters trade height for velocity and velocity for height. Most all calculations rely on using velocity measurements in one way or another. The first step is being able to calculate the changes in speed.

In an ideal world, mechanical energy is conserved. Frictional forces are ignored in early design stages. (This document does not address the nuances of dealing with frictional forces.) Mechanical energy on a roller coaster comes in two basic forms. Kinetic energy, $KE = (1/2)mv^2$, and potential energy, $PE = mgh$, due to gravity. Total energy, ET, is conserved and is equal to the sum of kinetic and potential at any single location.

ET = KE + PE (at any single location)



All four of these roller coasters look different. But because friction is ignored all four roller coasters will have the same speed at the bottom of each hill and at the top of the second. The only thing that matters is the height of the locations of interest.

Calculation Algorithm to calculate a change in velocity associated with a change in height

Step 1 Identify two locations of interest. One with both a speed and a height and the other location with either speed or height.

Step 2 Write an equation setting the total energy at one location equal to the total energy at the other location.

Step 3 Solve for the unknown variable.

Example 1

What is the velocity at the bottom of the first hill?

Solution:

$$ET_{(TOP)} = ET_{(BOTTOM)}$$

$$KE + PE = KE + PE$$

$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$

$$(1/2)v^2 + gh = (1/2)mv^2 + mgh$$

The masses cancel out because it is the same coaster at the top and bottom.

$$(1/2)v^2 + gh = (1/2)v^2 + gh$$

Substitute the numbers at each location

$$(1/2)(8.8)^2 + 9.8(95) = (1/2)v^2 + 9.8(0)$$

The height at the bottom is zero because it is the lowest point when comparing to the starting height.

$$77.44 + 931 = (1/2)v^2$$

$$1008.44 = (1/2)v^2$$

$$2016.88 = v^2$$

$$\underline{v = 44.9}\ m/s$$ ...at the bottom the the 1st hill.

Example 2

What is the velocity at the <u>top of the second hill</u>?

Solution:

$$ET_{(TOP\ OF\ 1^{st}\ HILL)} = ET_{(TOP\ OF\ 2^{nd}\ HILL)}$$

$$KE + PE = KE + PE$$

$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$

$$(1/2)v^2 + gh = (1/2)mv^2 + mgh$$

The masses cancel out because it is the same Coaster at the top and bottom.

$$(1/2)v^2 + gh = (1/2)v^2 + gh$$

Substitute the numbers at each location

$$(1/2)(8.8)^2 + 9.8(95) = (1/2)v^2 + 9.8(65)$$

Notice all the numbers on the left side come from the top of the 1st hill while all the numbers on the right side come from the top of the 2nd hill.

$$77.44 + 931 = (1/2)v^2 + 637$$

$$371.44 = (1/2)v^2$$

$$742.88 = v^2$$

$$\underline{v = 27.3}\ m/s$$ ...at the top the the 2nd hill.

This technique can be used to calculate the velocity anywhere along the coaster.

Getting the Coaster Started

(Work, Kinetic Energy, Potential Energy , Kinematics and Power)

Getting The Coaster Started

Something has to be done to get the coaster started. In our previous example energy, power, has to added get the coaster up to 8.8 m/s. This is done by doing work on the coaster. A simplified definition of work would be force times displacement when the force and displacement go in the same direction. [This chapter will not go into all the details of calculating work.] Suffice it to say that when the force acting on the coaster and the displacement of the coaster are in the same direction, work adds energy to the coaster. When the force acting on the coaster and the displacement of the coaster are in opposite directions, work removes energy from the coaster.

Energy is added to the car.

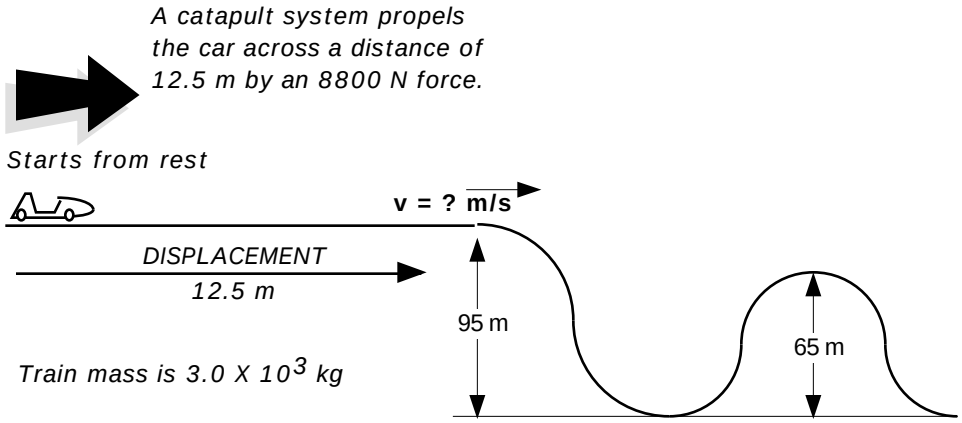
Energy is removed from the car.

$$\text{Work} = (\text{Force})(\text{Displacement})$$
$$W = Fd$$

Where "work" is measured in joules, J. "Force" is measured in Newtons, N, and "displacement" is measured in meters, m.

Example 1

What is the velocity of the train after being catapulted into motion?



Solution:

$$ET_{(\text{BEGINNING})} + \text{Work} = ET_{(\text{TOP OF } 1^{st} \text{ HILL})}$$
$$KE + PE + W = KE + PE$$
$$(1/2)mv^2 + mgh + Fd = (1/2)mv^2 + mgh$$
$$(1/2)3000(0)^2 + 3000(9.8)(3000) + 8800(12.5) = (1/2)3000v^2 + 3000(9.8)(0)$$

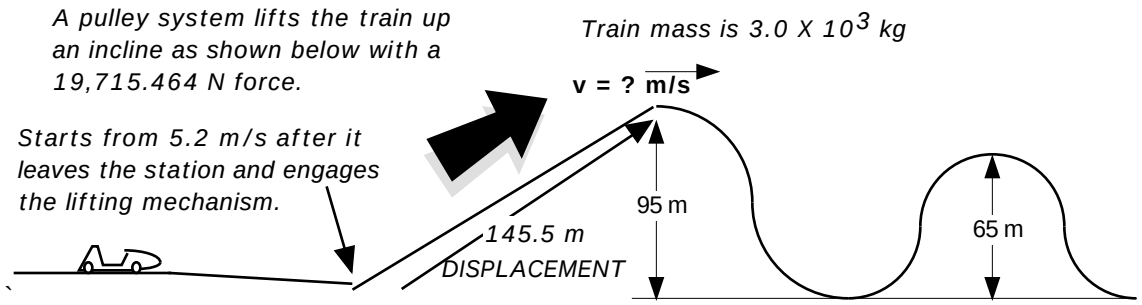
Substitute the numbers at each location

$$110{,}000 = (1/2)(3000)v^2$$
$$73.333 = v^2$$
$$v = 8.6\ m/s \quad \text{... at the end of the catapult.}$$

As an aside you can calculate the acceleration of the rider from kinematics equations. (For the curious the acceleration is 7.0 m/s^2)

Example 2

What is the velocity of the train after being catapulted into motion?



Solution:

$$ET_{(BEGINNING)} + Work = ET_{(TOP\ OF\ 1^{st}\ HILL)}$$
$$KE + PE + W = KE + PE$$
$$(1/2)mv^2 + mgh + Fd = (1/2)mv^2 + mgh$$
$$(1/2)3000(5.2)^2 + 3000(9.8)(0) + 19{,}715.464(145.5) = (1/2)3000v^2 + 3000(9.8)(95)$$
$$40560 + 2868600.012 = (1/2)(3000)v^2 + 2793000$$
$$116160.012 = (1/2)(3000)v^2$$
$$77.44 = v^2$$
$$v = 8.8\ m/s \quad \text{... at the end of the catapult.}$$

Example 3

A roller coaster train of mass 3.0 X 10^3 kg rolls over a 11.5 m high hill at 8.34 m/s before rolling down into the station. Once in the station, brakes are applied to the train to slow it down to 1.00 m/s in 5.44 m.

(a) What braking force slowed the train down?
(b) How much time did it take to slow the train down?
(c) What was the acceleration of the train in g's?

Solution:

(a)

$$ET_{(HILL)} = ET_{(@\ 1\ m/s)} + Work$$
$$KE + PE = KE + PE + W$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh + Fd$$
$$(1/2)3000(8.34)^2 + 3000(9.8)(11.5) = (1/2)3000(1)^2 + 3000(9.8)(0) + F(5.44)$$

Substitute the numbers at each location

$$442433.400 = 1500 + 5.44F$$

$\underline{F = 81053.9}$ N ...force to slow down the train

(b)

Calculate the velocity as the train enters the station. Use this velocity to calculate the time.
$ET_{(HILL)} = ET_{(@\ STATION\ ENTRANCE)}$ No work is done because no force acts between the two locations

$$KE + PE = KE + PE$$

$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$

$(1/2)3000(8.34)^2 + 3000(9.8)(11.5) = (1/2)3000v^2 + 3000(9.8)(0)$ Substitute the numbers at each location

$$442433.400 = 1500v^2$$

$$v = 17.174$$

$\underline{v = 17.2\ m/s}$ ...as the train enters the station.

The time is calculated from

$v_o = 17.174$ m/s

$v_f = 1.00$ m/s

$x = 5.44$ m

$t = ?$

$$\frac{x}{t} = \frac{v_o + v_f}{2}$$

$$\frac{5.44}{t} = \frac{17.174 + 1}{2}$$

$\underline{t = 0.599\ sec}$

(c)

Calculate the acceleration in m/s^2. Then convert it into g's.

$F = ma$

$81053.934\ N = (3000\ kg)a$

$a = 37.018\ m/s^2$

$a = 37.018\ m/s^2 / 9.80\ m/s^2 = \underline{3.78\ g's}$ ... Yeow! That's a big jerk on the passengers into the restraining harness.

Weightlessness

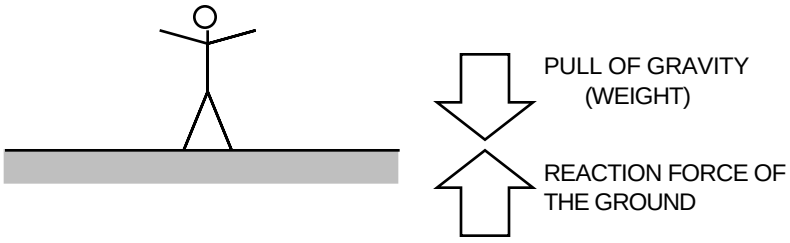
WEIGHTLESSNESS

Weight is the pull of gravity. Typical weight units are pounds and newtons. (1 pound 4.45 newtons). On the moon, gravity pulls with 1/6 the force compared to the Earth. Therefore, a student on the moon weighs 1/6 of what she weighs on the Earth.

On the Earth, neglecting air resistance, all objects will speed up at a rate of 9.80 m/s every second they fall. That is a speed increase of about 22 mph for every second an object falls.

Time in the Air *s*	*Velocity* *mph*
0	*0*
1	*22*
2	*44*
3	*66*
4	*88*
5	*110*

There are two ways to experience weightlessness. (1) move far enough away from the planets and sun to where their pull is nearly zero. [Gravity acts over infinite distance. One can never completely escape it.] (2) Fall down at a rate equal to the pull of gravity. In other words, accelerate to the Earth speeding up 22 mph every second in the air. In order for a person to feel weight, a person must sense the reaction force of the ground pushing in the opposite direction of gravity.



In the absence of the reaction force a person will sink through the ground.

Many amusement park rides generate the weightless sensation by accelerating down at 22 mph every second.

g's

Neglecting air resistance, if a rock is dropped, it will accelerate down at 9.8 m/s^2. This means it will speed up by 9.8 m/s for every second it falls. If a rock you drop accelerates down at 9.8 m/s^2, scientists say the rock is in a "1 g" environment, [1 g = 9.8 m/s^2 = 22 mph/s].

Any time an object experiences the pull equal to the force of gravity, it is said to be in a "one g" environment. We live in a 1 g environment. If a rock whose weight on the Earth is 100 lbs was moved to a 2 g environment then it would weigh 200 lbs. In a 9 g environment it would weigh 900 lbs. In a "NEGATIVE 2 g" environment it would take 200 lbs to hold the rock down on the ground. In a "-5 g" environment it would take 500 lbs to hold the rock down to the ground. If the rock were put into a "zero g" environment then it would be weightless. However,

no matter what happens to its weight the rock's mass would never change. Mass *measurement* is unaffected by the pull of gravity.

What does it feel like to walk in a 2 g environment? Have students find someone who's mass is about equal to theirs. Have them give piggyback rides. As they walk around this is what it feels like to be in a 2 g environment. Go outside on the soft ground and have the students step up on something. This is when they will really know what a 2 g environment feels like.

Often engineers will use g's as a "force factor" unit. The force factor gives a person a way of comparing what forces feel like.

All acceleration can be converted to g's by dividing the answer, in m/s^2, by 9.8 m/s^2.

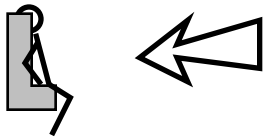
Example

A roller coaster is propelled horizontally by a collection of linear accelerator motors. The mass of the coaster train is 8152 kg. The train starts from rest and reaches a velocity of 26.1 m/s, 55 mph, in 3.00 seconds. The train experiences a constant acceleration. What is the coaster train's acceleration in g's?

Solution

m = 8152 kg
v_o = 0 (starts from rest)
v_f = 26.1 m/s
t = 3.00 s
a = ?

$v_f = v_o + at$
26.1 = 0 + a(3.00)
a = 8.70 m/s
in g's... 8.70 m/s^2 / 9.80 m/s^2 = <u>0.89 g's</u>
This means the rider is being pushed back into his seat by 89% of his weight.



The rider is pushed back into the seat by a force equal to 89 % of his weight.

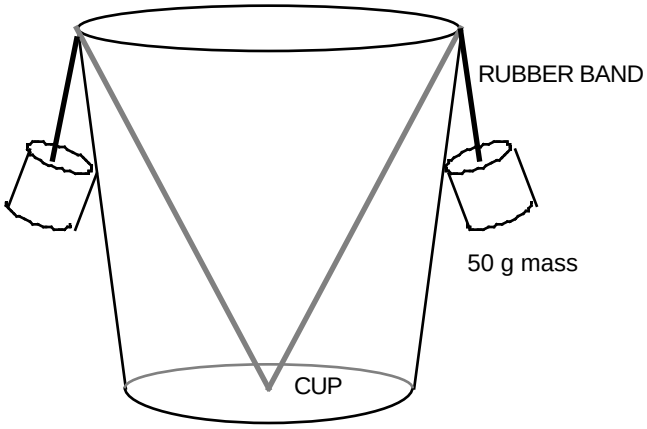
WEIGHTLESS DEMO #1

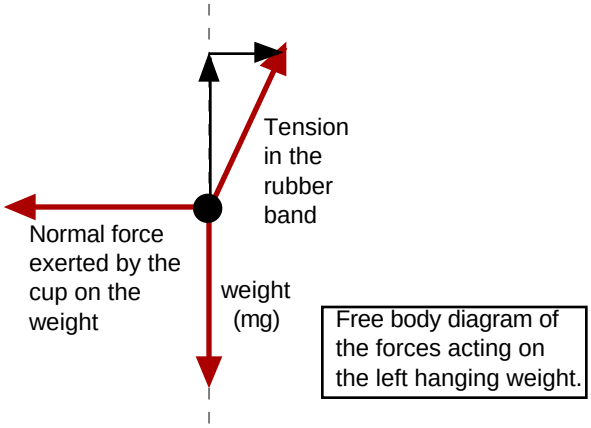
MATERIALS

1 plastic cup
2 skinny 4" diameter rubber bands
2 50 g masses

BACKGROUND

Cut the rubber bands. Tie the ends of each together to make a stretchy string. Tie the weights to the opposite ends of the rubber band. Attach the middle of the rubber band to the inside bottom of the cup. The two masses should be able to hang over the lip of the cup.

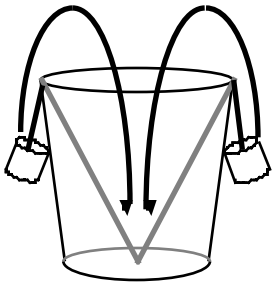




Free body diagram of the forces acting on the left hanging weight.

The masses are in equilibrium with the upward force of the rubber band. The force pulling up of the rubber bands is equal to the force of gravity, [weight of the masses.] Ask the class, "What would happen if the rubber bands pulled with a force greater than the pull of gravity on the masses?" The masses would shoot upward and be pulled into the cup. To show this, pull down on one of the masses and let go.

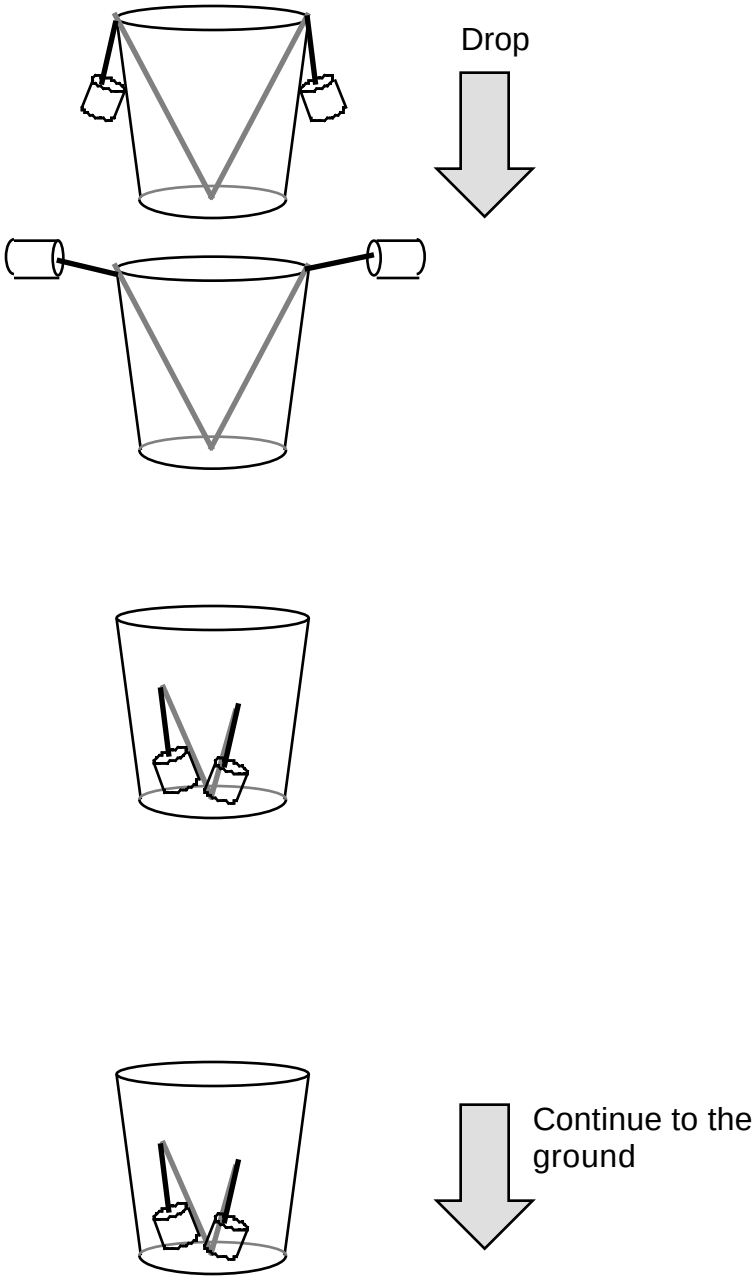
Now ask, "What would happen if the masses could be magically shielded from the pull of gravity?" With no force stretching the rubber bands, they would sling shot into the cup.



Explain, "We cannot yet shield gravity. But we can momentarily minimize its effects by accelerating the masses, rubber band and cup down at 9.80 m/s^2, the acceleration due to gravity. Without saying anymore, stand on a chair. Raise the apparatus with the weights hanging out. Tell the students, "When this cup is dropped everything will speed up equal to the acceleration of gravity. What will you see when this cup is dropped?"

Drop the cup after polling the students. The masses will be pulled into the cup. When everything falls, gravity will not be pulling against the masses when compared to the

rubber band's pulling force. The masses are said to be weightless. It is the weight of the mass that stretched the rubber band. If the mass is weightless, the rubber band will pull it in.



When everything is falling together, the pull of gravity is no longer experienced by the rubber bands. Therefore, they pull the masses into the cup.

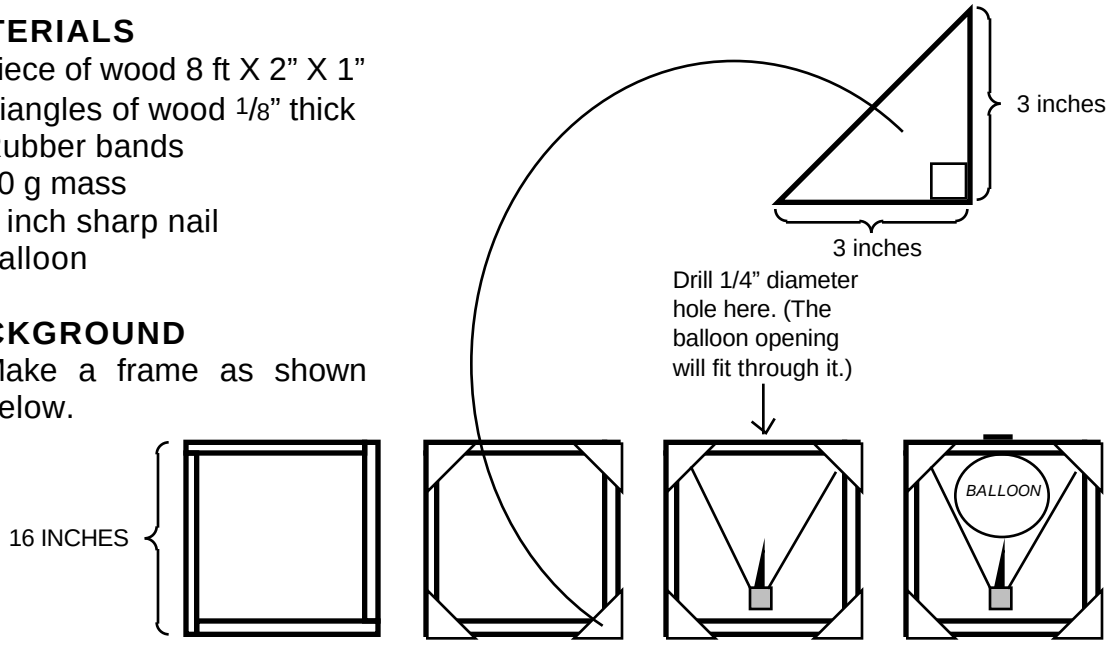
WEIGHTLESS DEMO #2

MATERIALS

1 piece of wood 8 ft X 2" X 1"
8 triangles of wood 1/8" thick
2 Rubber bands
1 20 g mass
1 3 inch sharp nail
1 balloon

BACKGROUND

Make a frame as shown below.



STEP 1
Nail it together.

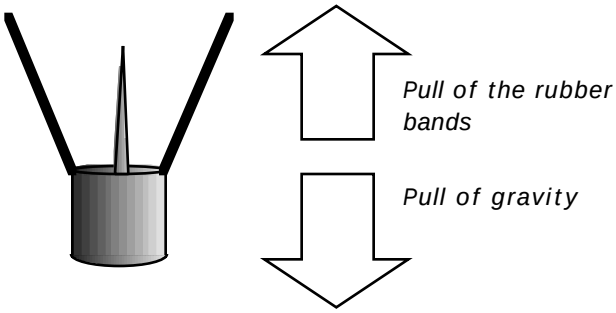
STEP 2
Glue and nail the triangles to the outside.

STEP 3
Attach the nail to the weight and hang the weight by rubber bands.

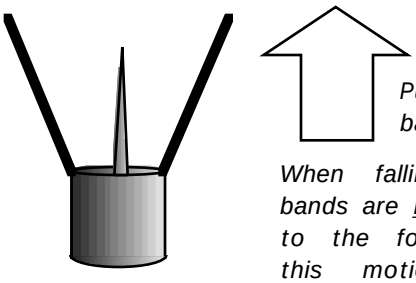
STEP 4
Insert the balloon in the hole. Inflate it so that it does not touch the nail. Tie it closed

The DEMO

With the balloon inflated, hold the frame over a pillow. Hold the frame straight out at chest height. Ask students to predict what will happen when you release the frame. Guide them to specifics such as where in the fall will the balloon pop. Release the frame. The balloon will pop almost instantaneously. The balloon pops because the weighted pin becomes weightless. The rubber bands are essentially pulling against nothing. This means the rubber bands pull the pin up into the balloon.



Forces acting on the mass when it is at rest.



When falling, the rubber bands are moving down due to the force of gravity. this motion negates the pull of gravity on the hanging mass. The rubber bands can now pull the hanging mass up -because it is weightless in free fall

WEIGHTLESS DEMO #3

MATERIALS

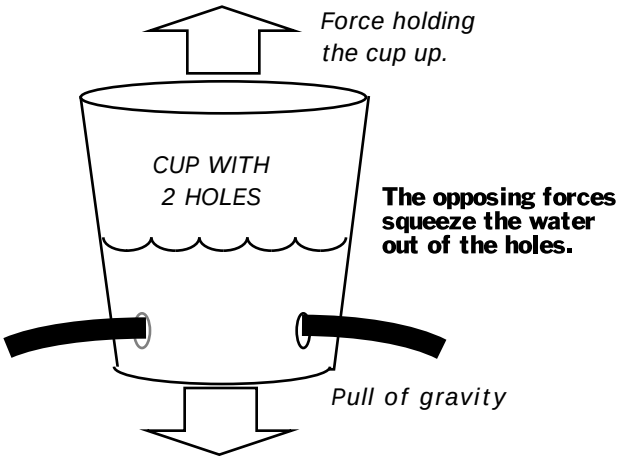
1 plastic cup
1 trash can or big bucket
1 candle flame large nail
1 large nail
- water

PREPARATION

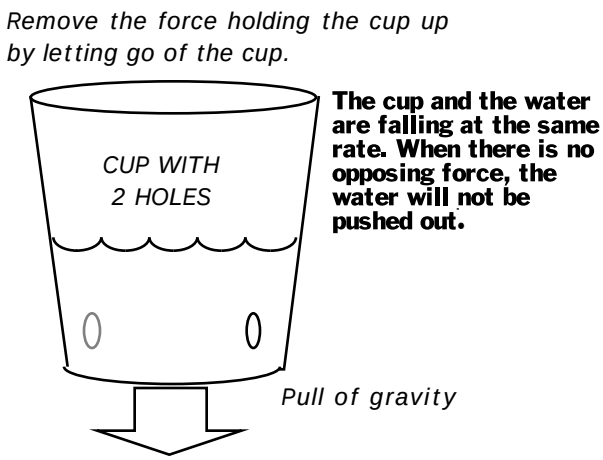
Heat up the nail with the candle flame. Be careful not to burn yourself. Poke the hot nail into opposite sides of the cup at the bottom. This will make a clean hole. Hold your fingers over the two holes and fill the cup half full of water.

THE DEMO

Stand on a chair and briefly release your fingers from the holes. The water should stream out the cup's holes onto the floor. Ask the students, "What will happen when I drop the cup into the trash can?" Listen to all their answers. Drop the cup to see who's prediction was correct. The water will not flow out of the cup. Water flows out of the cup when the acceleration of the cup is different from the water. When the cup is held the water is allowed to accelerate down at 9.8 m/s^2. When the cup falls too, the cup is also accelerating down at 9.8 m/s^2. Since there is no difference in their accelerations the water stays in the cup.



The water is accelerating down faster than the the cup. (That is because the cup is not accelerating at all.)



The water is <u>not</u> falling down quicker than the cup, so it stays in the cup.

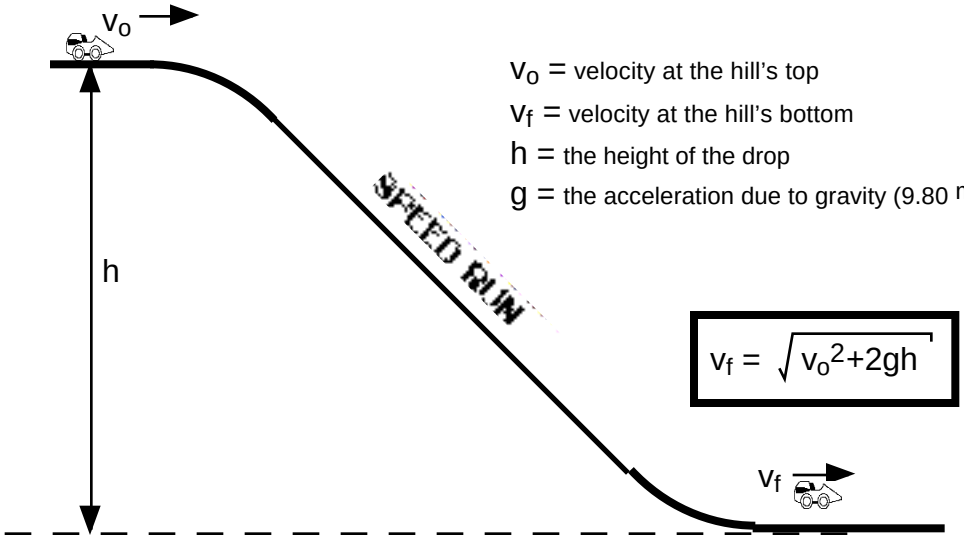
Hills and Dips

(Projectile Motion, Potential Energy and Kinetic Energy)

HILLS AND DIPS

One of the most basic parts of a ride is going from the top of a hill to the bottom. There are two basics ways designs transport riders to the bottom of a hill. The first is called the "Speed Run."

SPEED RUN DROPS



v_o = velocity at the hill's top

v_f = velocity at the hill's bottom

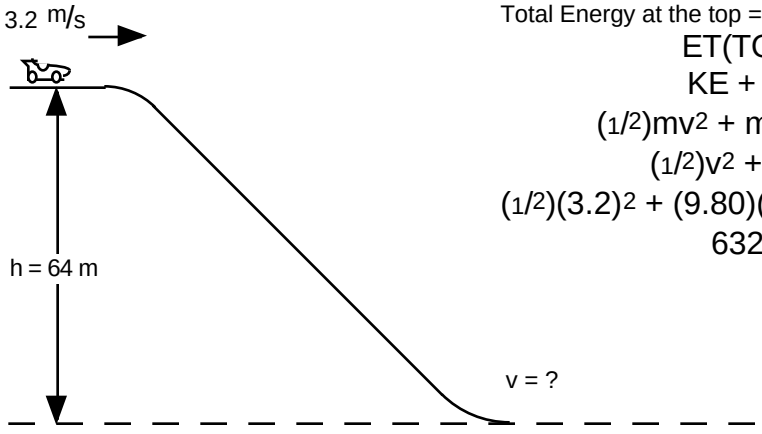
h = the height of the drop

g = the acceleration due to gravity (9.80 m/s^2)

$$v_f = \sqrt{v_o^2 + 2gh}$$

A speed run is designed to give the rider the feeling of accelerating faster and faster without the feeling of weightlessness. It simulates being in a powerful car with the accelerator held down to the floor. It is a straight piece of track that connects a high point to a low point.

The increase in velocity of the car comes from lost gravitational potential energy being converted into kinetic energy. Next to a horizontal straight piece, the speed run is the easiest piece of the track to design and analyze.



Total Energy at the top = Total Energy at the bottom

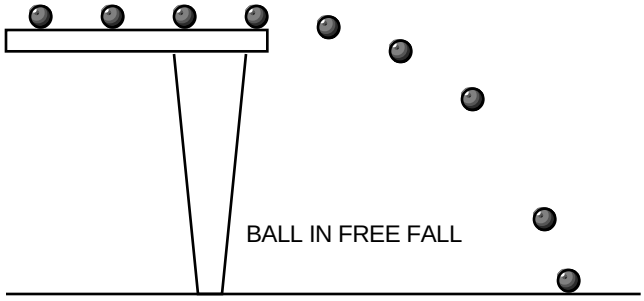
$$ET(TOP) = ET(BOTTOM)$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$$
$$(1/2)v^2 + gh = (1/2)v^2 + 0$$
$$(1/2)(3.2)^2 + (9.80)(64) = (1/2)v^2 + 0$$
$$632.32 = (1/2)v^2 + 0$$
$$v = 35.56$$
$$v = 36\ m/s$$

When coasting up to a new height the calculations are the same as the example shown above. The shape of the hill does not matter. See the "Intro to Design" section, step 7, for an example of these up hill calculations.

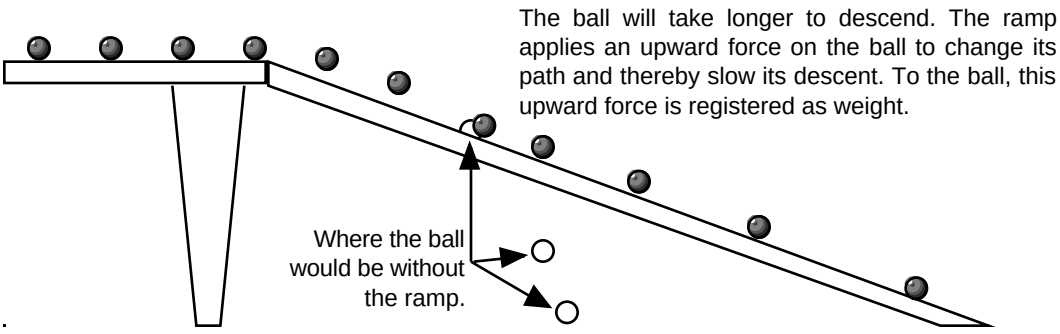
FREE FALL DROPS

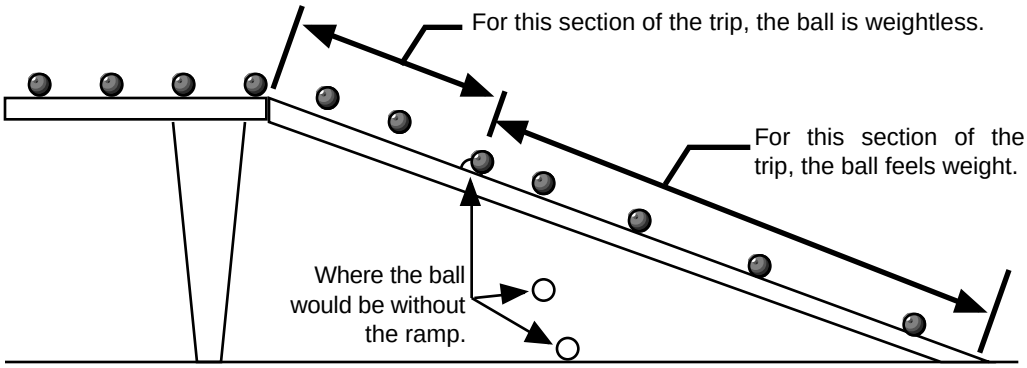
One of the biggest thrills on a roller coaster is the free fall as a rider travels over a hill. The easiest way to experience free fall is to hang from a tall height and drop to the ground. As a person falls he experiences weightlessness. As long as a person travels in the air like a projectile he will feel weightless.

Suppose a ball traveled off a table, horizontally, at 10 m/s. The ball's path would look like the path shown below.

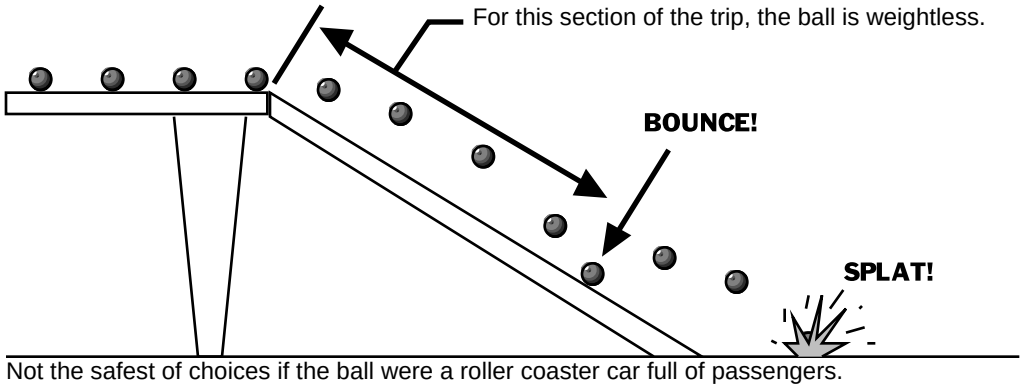


Now suppose the ball traveled off the table top on a shallow angled ramp. It would look like the one below.



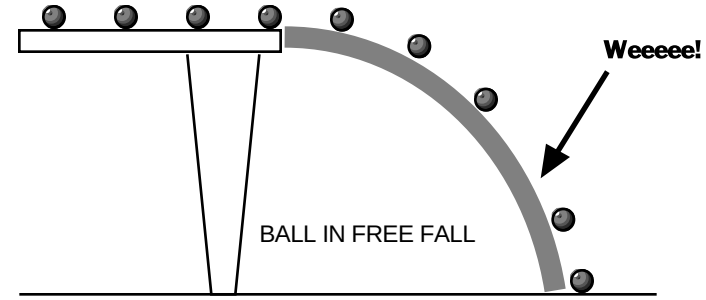


A straight, “speed run,” drop does not match the fall of a rider over a hill.



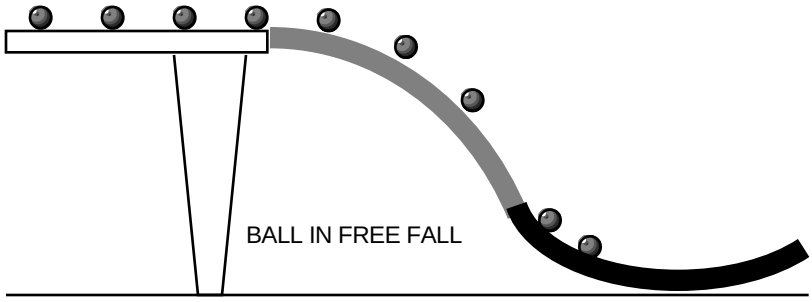
Not the safest of choices if the ball were a roller coaster car full of passengers.

To give riding more of a thrill, the designer needs to design the shape of the hill to match the falling ball.



The hill is the same shape as a projectile in free fall. The roller coaster barely makes contact with the track.

The only problem with curve above is the impact with the floor. To alleviate this problem another curve scoops the balls as they descend. This makes the ride smooth and survivable for the rider.



The speed at the bottom of a free fall drop is calculated the same way as the speed at the bottom of a speed run drop. The only difference is the shape of the hill from the top to the bottom.

PROJECTILE MOTION AND ROLLER COASTER HILLS

A free fall hill shape gives a rider a weightless sensation. To give this weightless sensation over a hill, the hill is designed to have the same shape as the path of a ball being thrown off the top of a hill. Shape is determined by how fast the roller coaster car travels over the hill. The faster the coaster travels over the hill the wider the hill must be. There are two ways to apply projectile motion concepts to design the hill's shape. The first way is to calculate the coaster's position as if it drove off a cliff.

The position equation is as follows.

$$h = \frac{gx^2}{2v^2}$$

h = the height from the top of the hill
x = is the distance away from the center of the hill
v = the velocity the roller coaster car travels over the top of the hill.
g = the acceleration due to gravity. 9.80 m/s^2 for answers in meters. 32.15 ft/s^2 for answers in feet.

This can be rewritten as

$$x = \sqrt{\frac{2hv^2}{g}}$$

EXAMPLE CALCULATIONS

Velocity (v) = 10 m/s
Acceleration (g) = 9.8 m/s^2

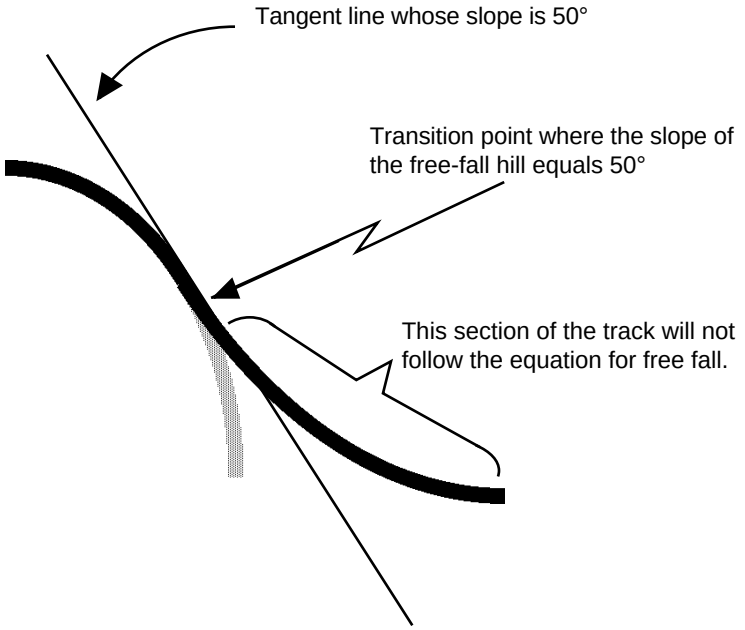
x in meters	*h in meters*
0.00	*0*
4.52	*1*
6.39	*2*
7.82	*3*
9.04	*4*
10.10	*5*
11.07	*6*
11.95	*7*
12.78	*8*
13.55	*9*
14.29	*10*

Velocity (v) = 20 m/s
Acceleration (g) = 9.8 m/s^2

x in meters	*h in meters*
0.00	*0*
9.04	*1*
12.78	*2*
15.65	*3*
18.07	*4*
20.20	*5*
22.13	*6*
23.90	*7*
25.56	*8*
27.11	*9*
28.57	*10*

Catching the Rider

There comes a certain point on the free-fall drop where the track needs to redirect the riders. Otherwise the riders will just plummet into the ground. This point is the transition point from free-fall to controlled acceleration. This point is also the maximum angle of a hill. This angle can be in virtually any range from 35° to 55°.



For the bottom section of the track, the new equation has the desired outcome of changing the direction of the coaster from a downward motion to a purely horizontal motion. The track will need to apply a vertical component of velocity to reduce the coaster's vertical velocity to zero. The track will also need to increase the horizontal velocity of the coaster to the value determined from energy relationships. The velocity at the bottom of the hill is determined from

$$\underbrace{(\text{Kinetic Energy}) + (\text{Gravitational Potential Energy})}_{\text{Total mechanical energy at the top}} = \underbrace{(\text{Kinetic Energy})}_{\text{Total mechanical energy at the bottom}}$$

which is

$$(1/2)m(v_T)^2 + (mgh) = (1/2)m(v_B)^2$$

This simplifies to

$$v_B = \sqrt{(v_T)^2 + 2gh}$$

where v_B is the horizontal velocity at the bottom of the hill. The value for v_B will be used in later calculations.

Recall one of the original horizontal equations.

$$x = x_o + (v_{xo})t + (1/2)(a_x)t^2$$

substituting in our expression for "t" yields,

$$x = (v_{xo})\left(\frac{\sqrt{v_{yo}^2 + 2(a_y)y} - v_{yo}}{(a_y)}\right) + \frac{a_x}{2}\left(\frac{\sqrt{v_{yo}^2 + 2(a_y)y} - v_{yo}}{(a_y)}\right)^2$$

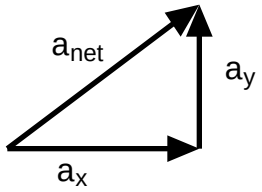
where v_{xo} is the horizontal velocity of the coaster at the transition angle and v_{yo} is the vertical component of the velocity at the transition angle. "a_y" is calculated from

$$v_y^2 = v_{yo}^2 + 2(a_y)y$$

and

$$(a_y) = \frac{v_y^2 - v_{yo}^2}{2y}$$

Where "v_y" is the final vertical velocity of zero, "v_{yo}" is the vertical component of the velocity at the transition point, and "y" is the distance left to fall from the transition point to the ground. The horizontal velocity is determined from a parameter decided upon by the engineer. The engineer will want to limit the g forces experienced by the rider. This value will be the net g's felt by the rider. These net g's are the net acceleration.



$$a_{net}^2 = a_x^2 + a_y^2$$

and

$$a_x = \sqrt{a_{net}^2 - a_y^2}$$

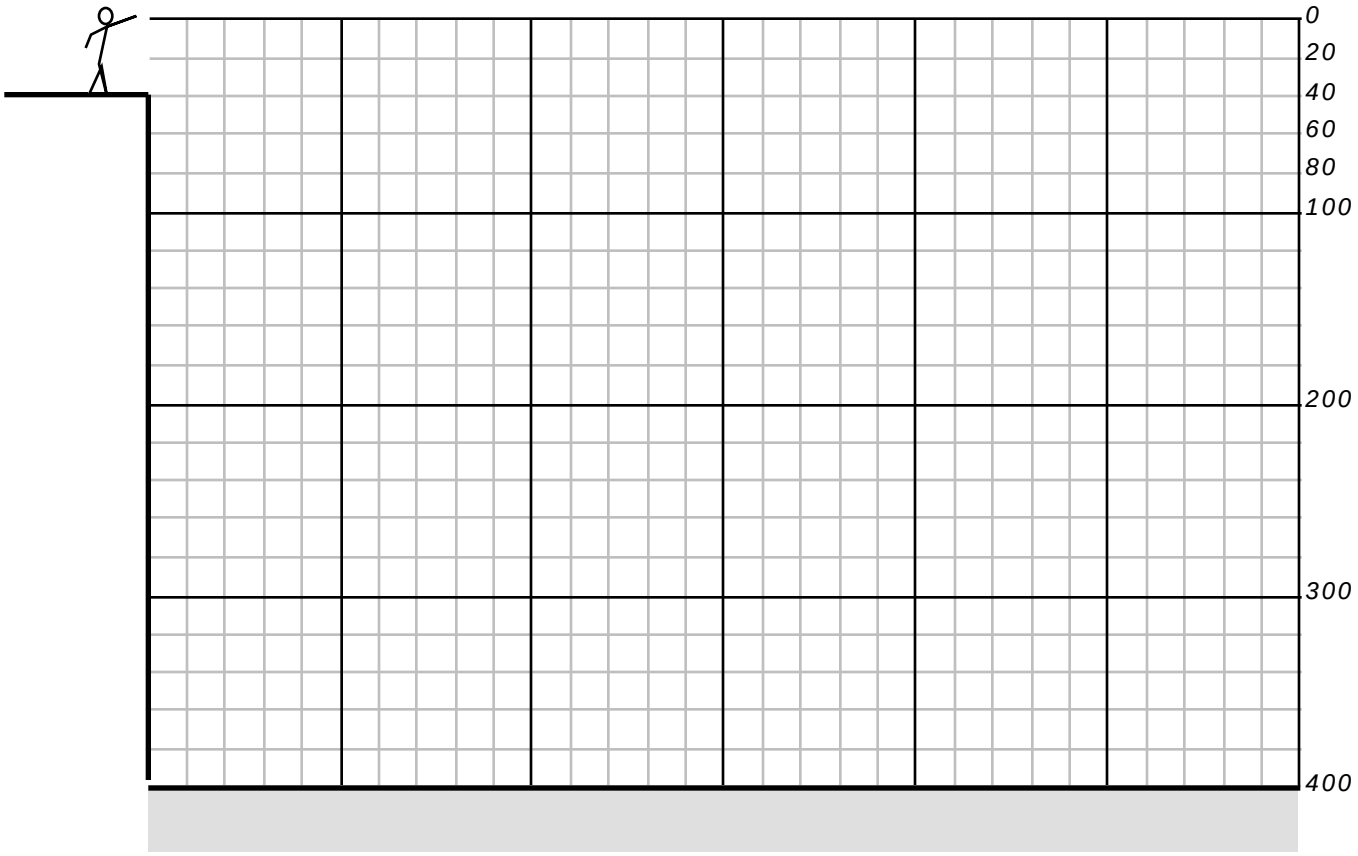
these values are plugged back into the original equation and x values are calculated as a function of y.

This is the beginning of the Hurler at Paramount's Kings Dominion in Doswell Virginia. Can you tell which hill is the free fall hill?

(It's the curved hill in front.)

Projectile Motion and Free Fall Hills Worksheet

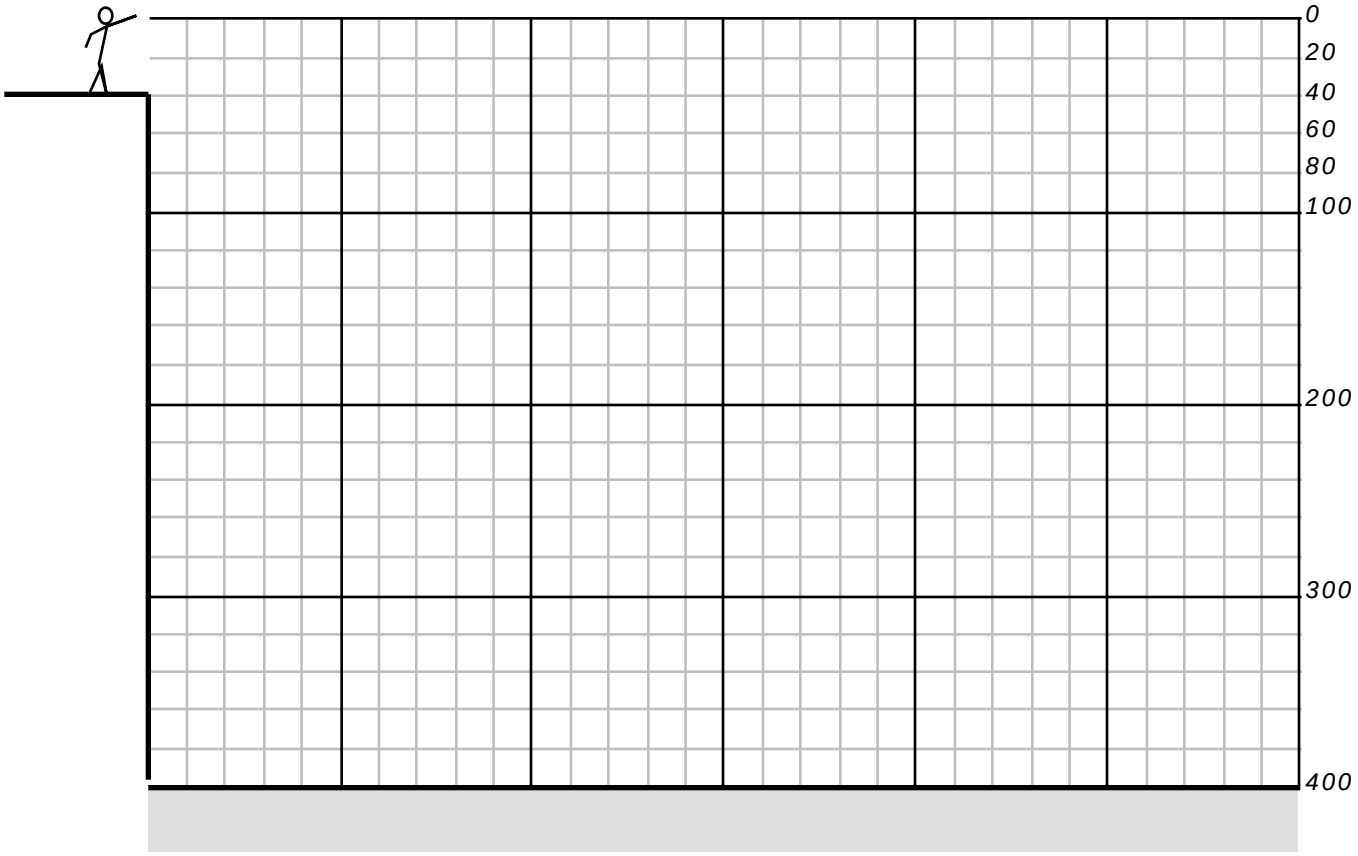
A person throws 2 balls. The first ball is thrown horizontally at 20 m/s. The second ball is thrown at 40 m/s. Draw as much of each path as possible. Draw the ball's position every 20 m of vertical flight. Draw a smooth line to show the curve's shape.



Vertical distance down (meters)	*Horizontal position for the ball with an initial velocity of 20 m/s*	*Horizontal position for the ball with an initial velocity of 40 m/s*
0	*0*	*0*
40		
80		
120		
160		
200		
240		
280		
320		
360		
400		

Projectile Motion and Hills Worksheet#2

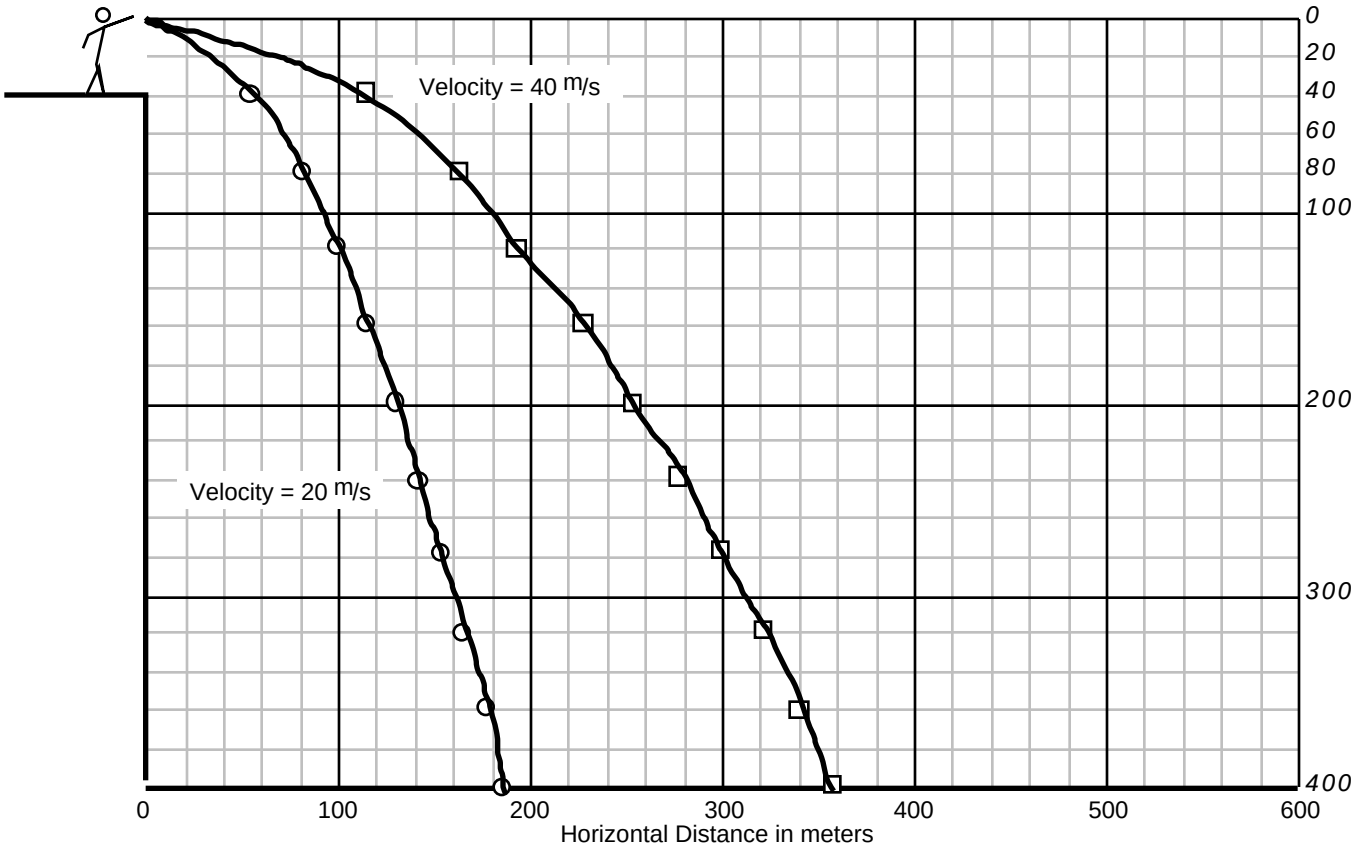
A person throws 2 balls. The first ball is thrown horizontally at 10 m/s. The second ball is thrown at 30 m/s. Draw as much of each path as possible. Draw the ball's position every 1 second of the flight. Draw a smooth line to show the curve's shape.



Time (seconds)	*Vertical position for the ball with an initial velocity of 10 m/s*	*Vertical position for the ball with an initial velocity of 30 m/s*
0	*0*	*0*
1		
2		
3		
4		
5		
6		
7		
8		
9		

Projectile Motion and Free Fall Hills Worksheet
ANSWERS

A person throws 2 balls. The first ball is thrown horizontally at 20 m/s. The second ball is thrown at 40 m/s. Draw as much of each path as possible. Draw the ball's position every 20 m of vertical flight. Draw a smooth line to show the curve's shape.



Below is a set of answers for 4 velocities.

Vertical distance down (meters)	*Horizontal position for the ball with an initial velocity of 10 m/s*	*Horizontal position for the ball with an initial velocity of 20 m/s*	*Horizontal position for the ball with an initial velocity of 30 m/s*	*Horizontal position for the ball with an initial velocity of 40 m/s*
0	*0*	*0*	*0*	*0*
40	*28.6*	*57.1*	*102.4*	*136.6*
80	*40.4*	*80.8*	*121.8*	*162.4*
120	*49.5*	*99.0*	*134.8*	*179.8*
160	*57.1*	*114.3*	*144.9*	*193.2*
200	*63.9*	*127.8*	*153.2*	*204.3*
240	*70.0*	*140.0*	*160.3*	*213.8*
280	*75.6*	*151.2*	*166.6*	*222.2*
320	*80.8*	*161.6*	*172.3*	*229.7*
360	*85.7*	*171.4*	*177.4*	*236.6*
400	*90.4*	*180.7*	*182.2*	*242.9*

ACTIVITY

Materials:

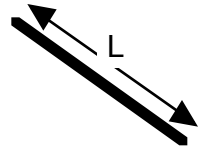
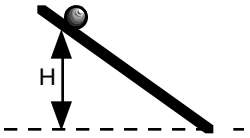
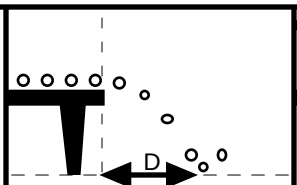
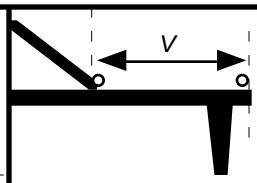
Ruler, marble, horizontal table

Objective:

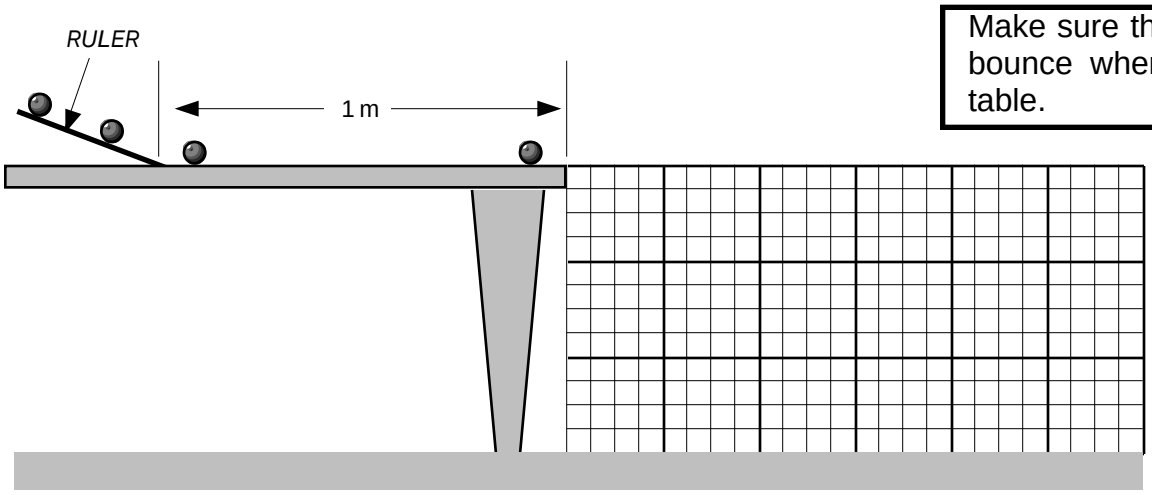
To see the effect of speed on arc shape.

Procedure:

Hold the end of the ruler up to three different angles. Repeatedly roll the ball down the ruler until you can draw the path the ball takes for each height. (Make sure the marble does not bounce when it leaves the table.) Write your data down for each trial in the data table below.

 Length of the ramp the ball rolls down.	 Height where the ball starts	 Distance to the ball's landing spot.	 Average velocity along the table top ONLY.
(m)	(m)	(m)	(m/s)
			$V_{avg} = {}^{dist}/_{time}$

Draw the path the ball takes on the grid on the next page for each ruler height.



Make sure the ball does not bounce when it leaves the table.

The drawn path represents the shape of a free fall roller coaster hill for different coaster car velocities.

Questions

1 Which ramp angle gives the ball the greatest speed when it leaves the table?

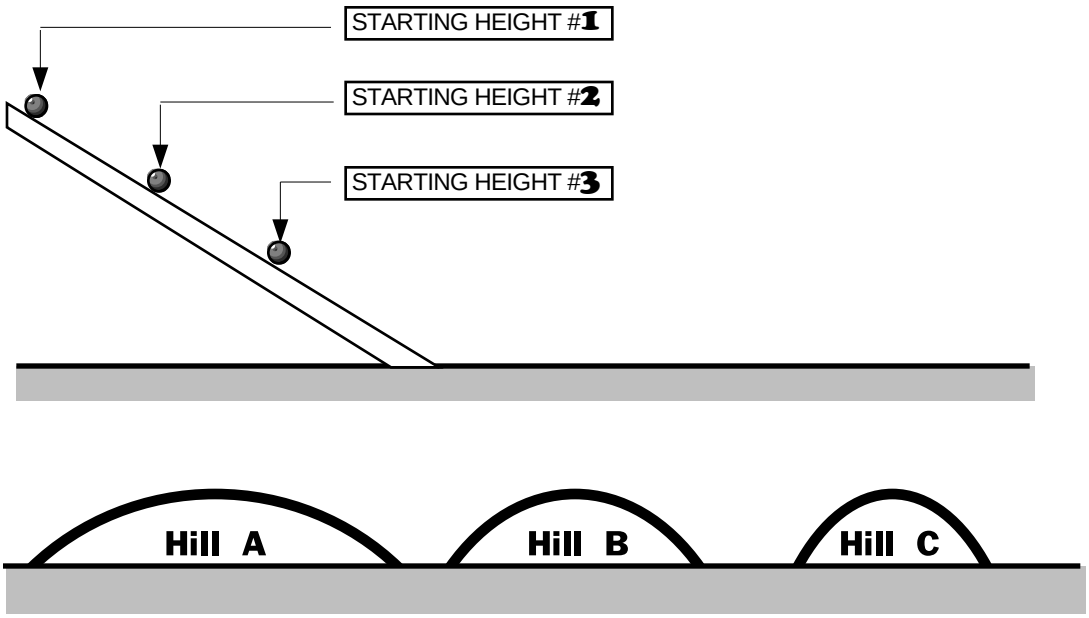
2 What conclusions can you make about how speed over the top of a hill affects the shape of the hill?

Below are three different starting heights for a hill. For each starting height, rate the hills shape as:

(1) Possible optimum hill. The ball will be in free fall the longest period of time and the ball would softly be caught at the bottom.

(2) Safe hill. The ball stays on the hill but does not remain in free fall for the maximum amount of time.

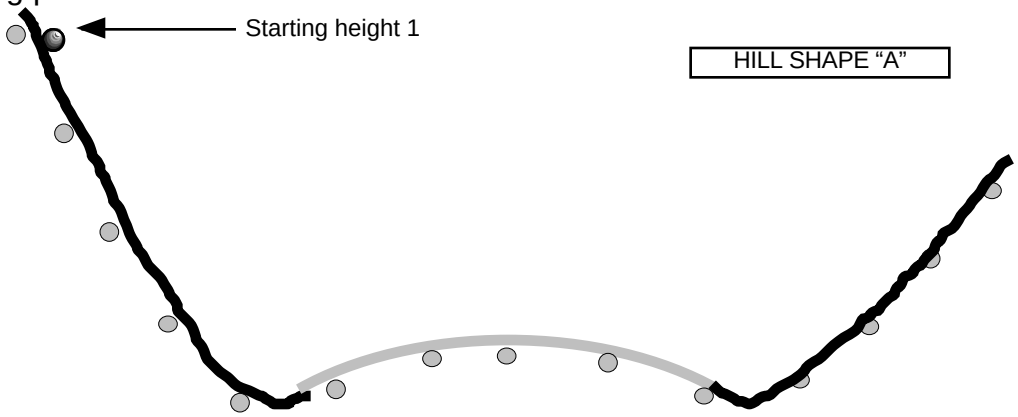
(3) Unsafe hill. The ball will leave the track and possibly hit the other side of the track.

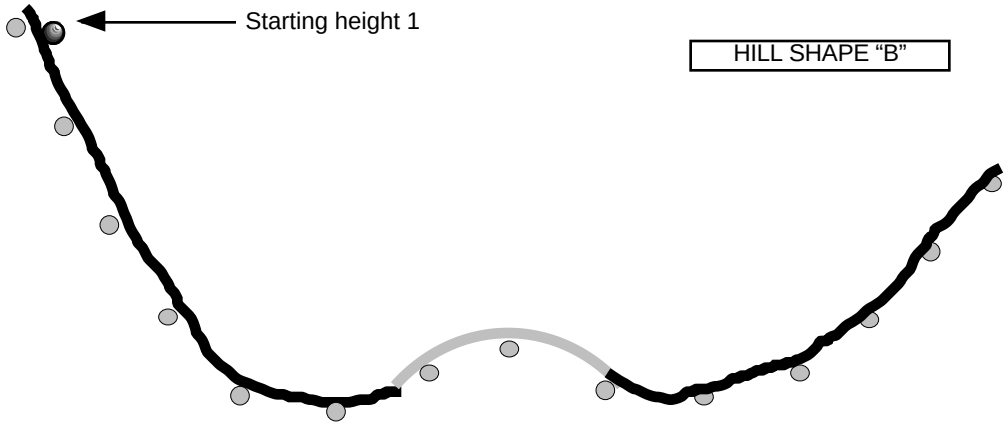


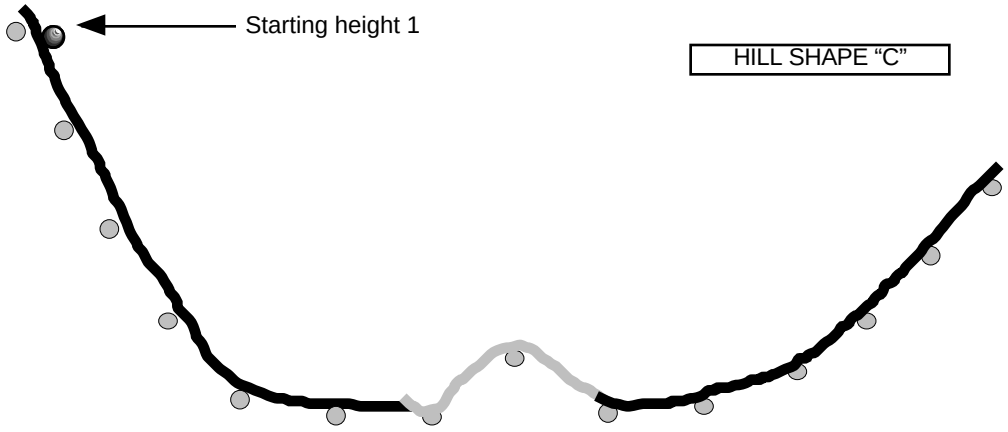
STARTING POSITION	HILL A	HILL B	HILL C
1			
2			
3			

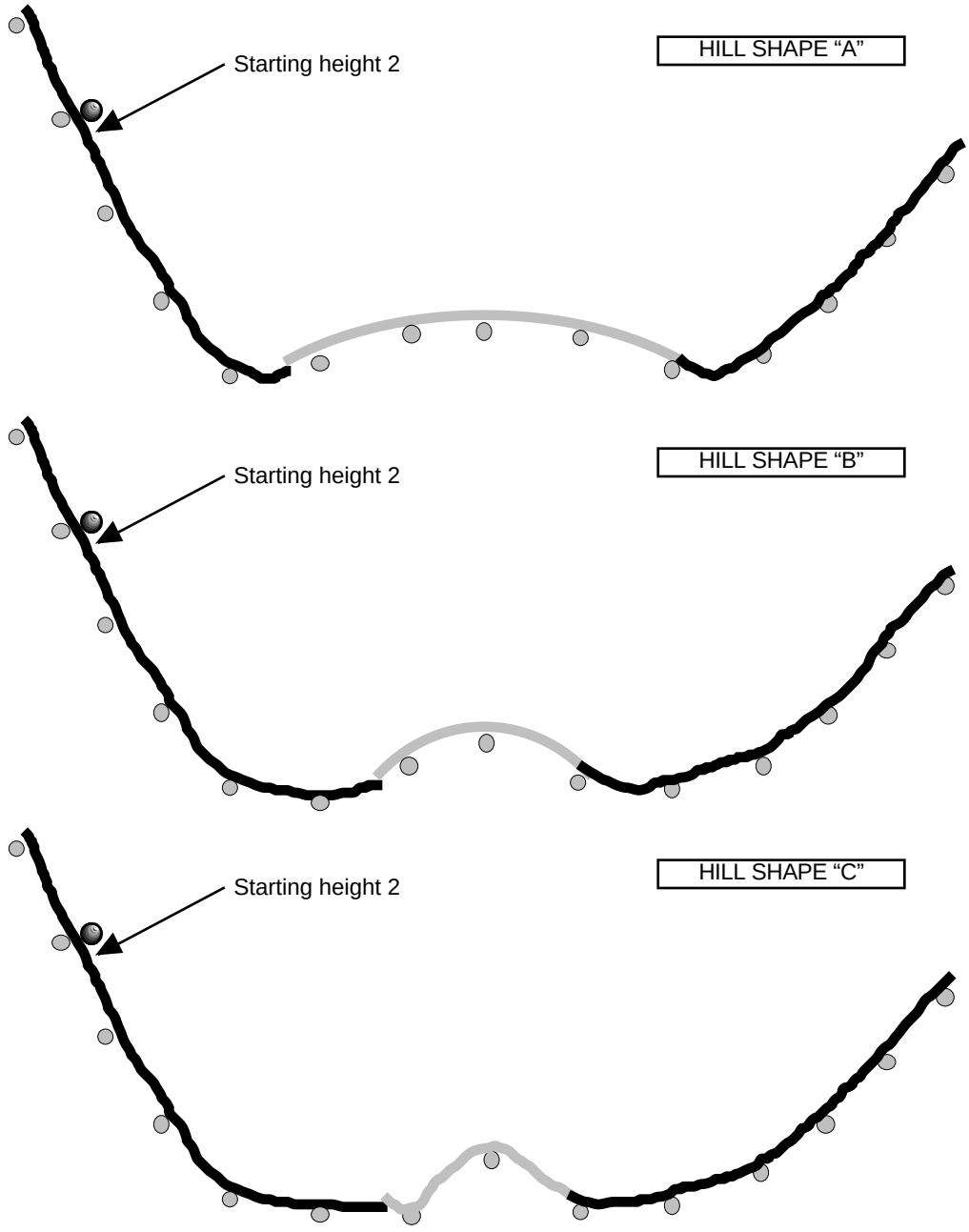
In this activity the student will visualize the path of a ball as it rolls over different shaped hills. In this activity a ball will be rolled from 3 different heights over three different shaped hills.

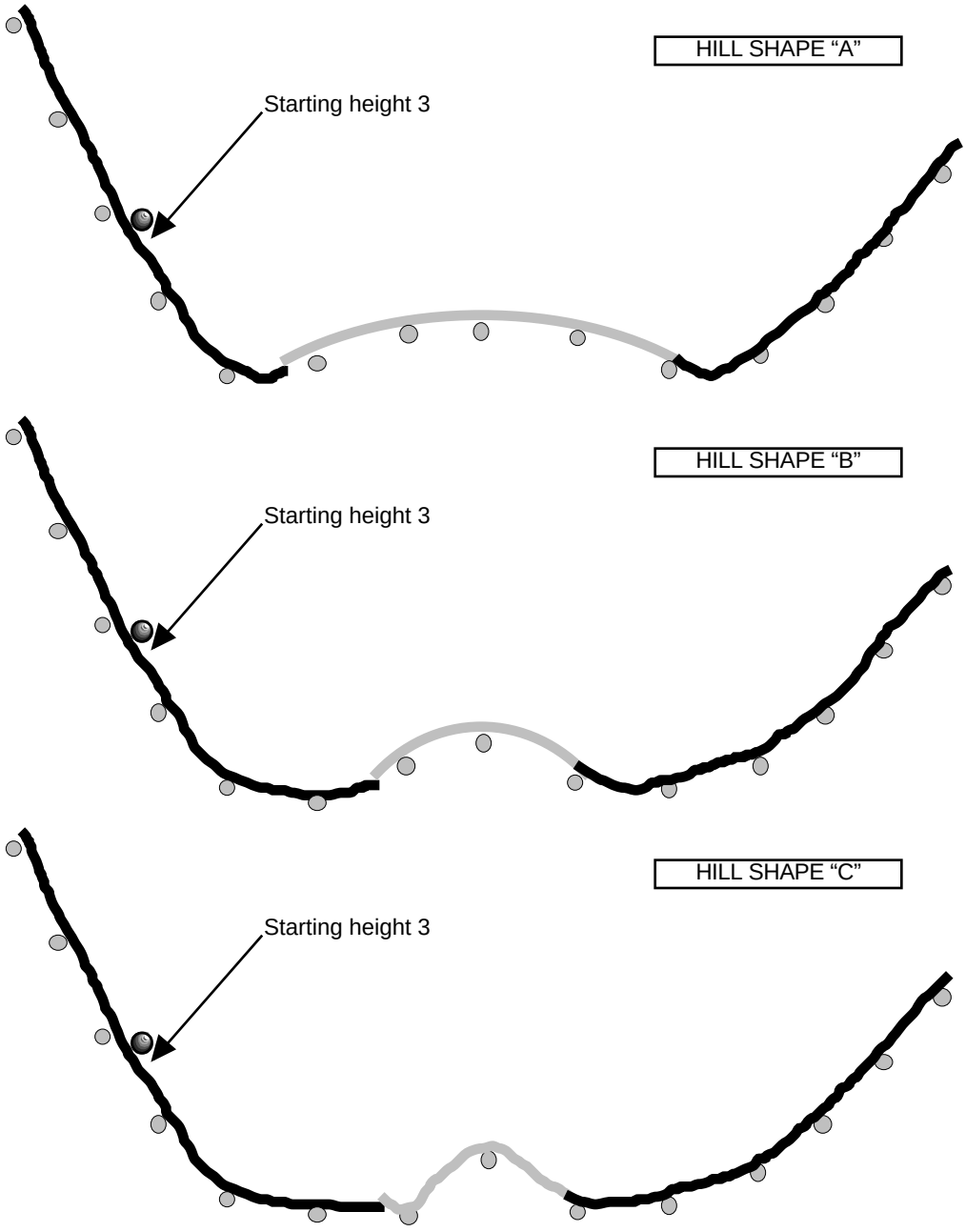
- Lay out the roller coaster simulator track on the chalk board.
- Using a piece of chalk trace the path of the track and mark the starting point of the track.
- Roll the ball down the track. If the ball leaves the track, trace its path with chalk. Then roll it again to see how it compares with the drawn line.
- Draw the path the ball takes on the paper. Repeat this process for every hill and every starting position.








Starting height 2
HILL SHAPE “A”
Starting height 2
HILL SHAPE “B”
Starting height 2
HILL SHAPE “C”

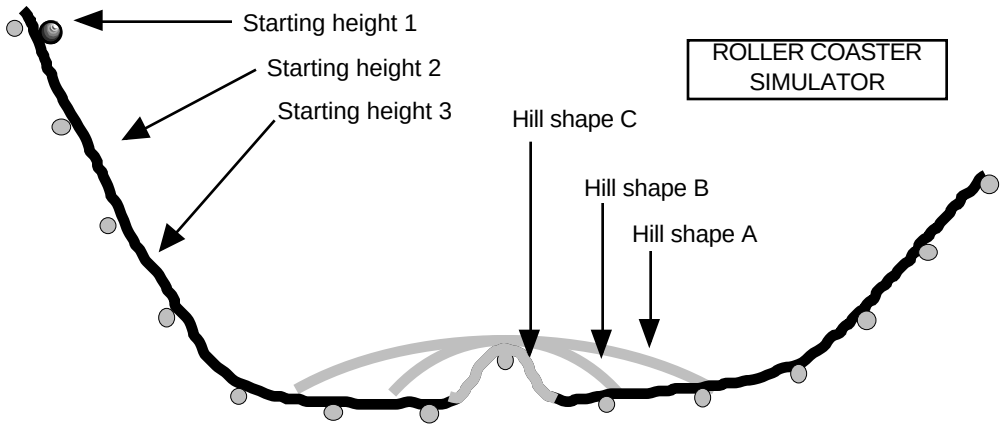

HILL SHAPE “A”
Starting height 3
HILL SHAPE “B”
Starting height 3
HILL SHAPE “C”
Starting height 3

Below are three different starting heights for a hill. For each starting position rate the hills shape as:

(1) Possible optimum hill. The ball will be in free fall the longest period of time and the ball would softly be caught at the bottom.

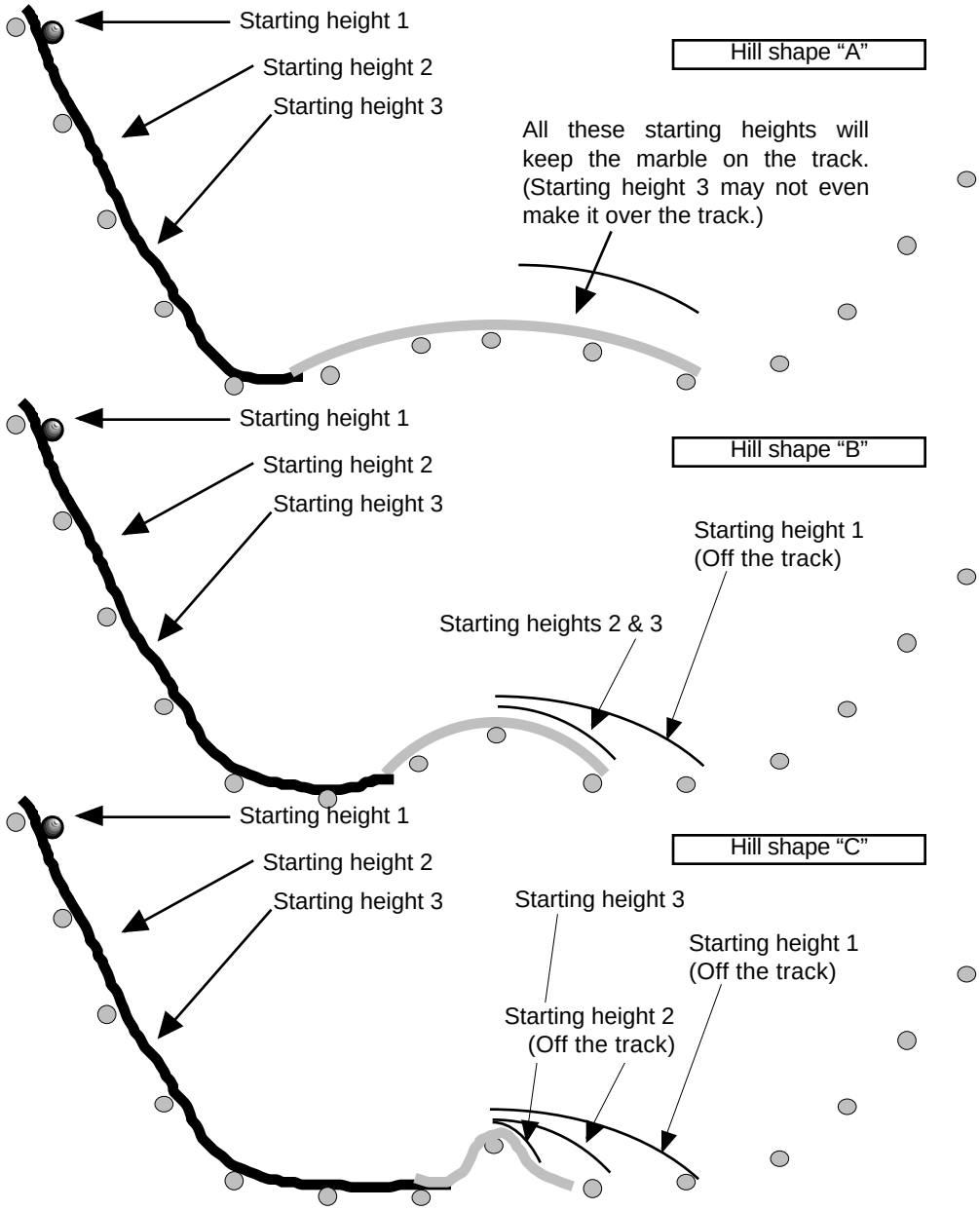
(2) Safe hill. The ball stays on the hill but does not remain in free fall for the maximum amount of time.

(3) Unsafe hill. The ball will leave the track and possibly hit the other side of the track.



STARTING POSITION	HILL A	HILL B	HILL C
1			
2			
3			

ANSWERS


Starting height 1
Starting height 2
Starting height 3
Hill shape “A”
All these starting heights will keep the marble on the track. (Starting height 3 may not even make it over the track.)
Starting height 1
Starting height 2
Starting height 3
Hill shape “B”
Starting height 1 (Off the track)
Starting heights 2 & 3
Starting height 1
Starting height 2
Starting height 3
Hill shape “C”
Starting height 3
Starting height 1 (Off the track)
Starting height 2 (Off the track)

ANSWERS

STARTING POSITION	HILL A	HILL B	HILL C
1	**OPTIMUM** The ball is the highest and will be traveling the fastest. A fast speed means a long "flat" hill.	**UNSAFE** The ball will get airborne over this hill.	**UNSAFE** The ball will get airborne over this hill.
2	**SAFE** The ball will make it over the hill. The ball will not experience weightlessness because the ball wants to fall quicker than the hill will allow.	**OPTIMUM** The ball is the highest and will be traveling the fastest. A fast speed means a long "flat" hill.	**UNSAFE** The ball will get airborne over this hill.
3	**SAFE** The ball will make it over the hill. The ball will not experience weightlessness because the ball wants to fall quicker than the hill will allow.	**SAFE** The ball will make it over the hill. The ball will not experience weightlessness because the ball wants to fall quicker than the hill will allow.	**OPTIMUM** The ball is the highest and will be traveling the fastest. A fast speed means a long "flat" hill.

Loops

(Circular Motion, Potential Energy and Kinetic Energy)